Warm Up

1. Determine the distance between points (−6, 0) and (4, 0).

   10 units

2. Determine the coordinates of the point that is located halfway between the points (−6, 0) and (4, 0).

   The point (-1, 0) is halfway between the points.

3. Determine the distance between points (−4, 15) and (12, 15).

   16 units

4. Determine the coordinates of the point that is located halfway between the points (−4, 15) and (12, 15).

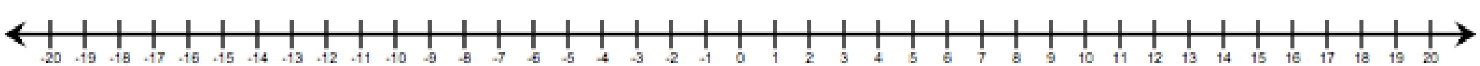
   The point (4, 15) is halfway between the points.

5. How did you determine the distance between the two points in Questions 1 and 3?

   Subtract the x-coordinates of each point from each other.

6. How did you determine the coordinates of the point located halfway between the two given points in Questions 2 and 4?

   Divide the distance between the two points by 2.



Just Watch that Pumpkin Fly!

Investigating the Vertex of a Quadratic Function

11.5

LEARNING GOALS

In this lesson, you will:

- Interpret parts of a quadratic function in terms of a problem situation.
- Use a calculator to determine the *x*-intercept(s), *y*-intercept, and absolute maximum or minumum of a quadratic function.
- Solve a quadratic function graphically.
- Determine the vertex of a quadratic function.
- Use symmetric points to determine the location of the vertex of a parabola.
- Use the vertex to determine symmetric points on a parabola.

KEY TERMS

- vertex
- axis of symmetry

PROBLEM 1 Punkin Chunkin by Catapult!

You can model the motion of a pumpkin released from a catapult using a vertical motion model. Remember, a vertical motion model is a quadratic equation that models the height of an object at a given time. The equation is of the form

$$y = -16t^2 + v_0t + h_0,$$

where y represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial vertical velocity (speed) of the object in feet per second, and h_0 represents the initial height of the object in feet.

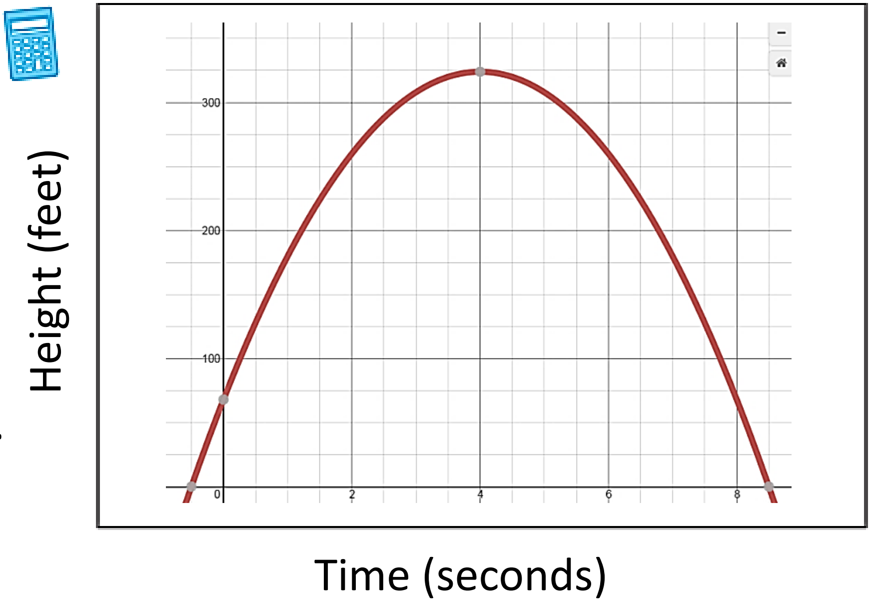
1. Why do you think it makes sense that this situation is modeled by a quadratic function?
   The motion of a catapulted pumpkin is very similar to the shape of a parabola.

Suppose that a catapult hurls a pumpkin from a height of 68 feet at an initial vertical velocity of 128 feet per second.

2. Write a function for the height of the pumpkin h(t) in terms of time t.
   $h(t) = -16t^2 + 128t + 68$

3. Does the function you wrote have an absolute minimum or an absolute maximum? How can you tell from the form of the function?
   Since the leading coefficient is negative, the parabola opens downward so the function has an absolute maximum.

4. Graph the function on Desmos.com.
Sketch your graph and LABEL the axes.
Change the settings so
$-1 \le x \le 9$ and $-50 \le y \le 500$



5. Use a graphing calculator to determine the zeros of the function. Then explain what each means in terms of the problem situation. Do each make sense in terms of this problem situation?

Zeros: (-0.5, 0) and (8.5, 0)

(-0.5, 0) means the pumpkin was on the ground 0.5 seconds before it was catapulted. There is no way to know if this is true so it does not make sense.

(8.5, 0) means that the pumpkin was on the ground or landed 8.5 seconds after it was catapulted, which makes sense.

6. Determine the y-intercept and interpret its meaning in terms of this problem situation.

Y-intercept: (0, 68). At 0 seconds, the pumpkin's initial height was 68 feet.

7. Use a graphing calculator to determine the absolute minimum or maximum . Then explain what it means in terms of this problem situation .

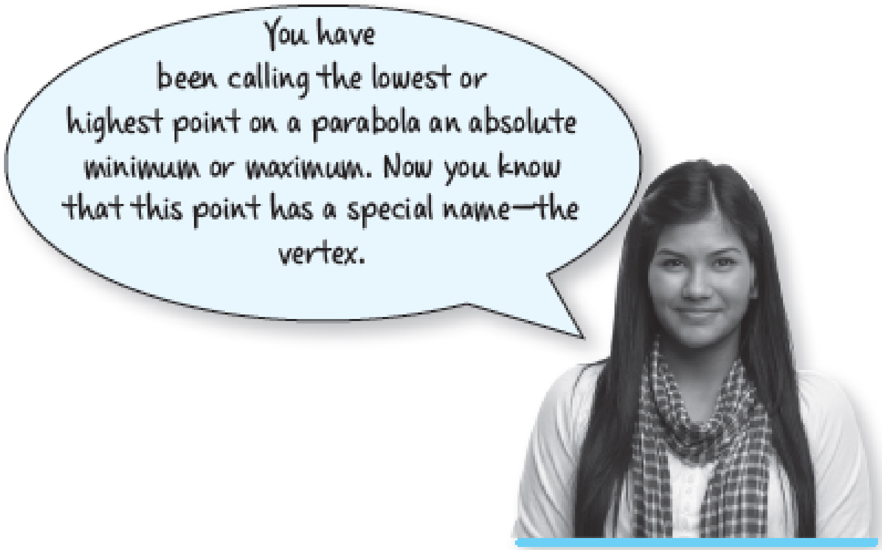
Absolute maximum: (4, 324)

The pumpkin reached a maximum height of 324 feet in 4 seconds.

PROBLEM 2 The Vertex of a Parabola and Symmetry

The **vertex** of a parabola is the lowest or highest point on the curve. The **axis of symmetry** of a parabola is the vertical line that passes through the vertex and divides the parabola into two mirror images.

Because the axis of symmetry always divides the parabola into two mirror images, you can say that a parabola is symmetric.



1. Because a parabola is symmetric, over which line would you fold it to show its symmetry? What is the equation of that line?

Axis of symmetry. Its equation is x = (the x-coordinate of the vertex).

Remember the Punkin Chunkin scenario in Problem 1, in which you wrote the function $h(t) = -16t^2 + 128t + 68$ for the height of the pumpkin in terms of time.

2. Identify the coordinates of the vertex of the graph and the equation for the axis of symmetry. (See Problem 1, #7.)

   Vertex: (4, 324)

   Axis of Symmetry: x = 4

3. Use a graphing calculator to answer each question.

   a. When does the pumpkin reach a height of 128 feet?

      0.5 seconds and again at 7.5 seconds

   b. When does the pumpkin reach a height of 180 feet?

      1 second and again at 7 seconds

   c. When does the pumpkin reach a height of 308 feet?

      3 seconds and again at 5 seconds

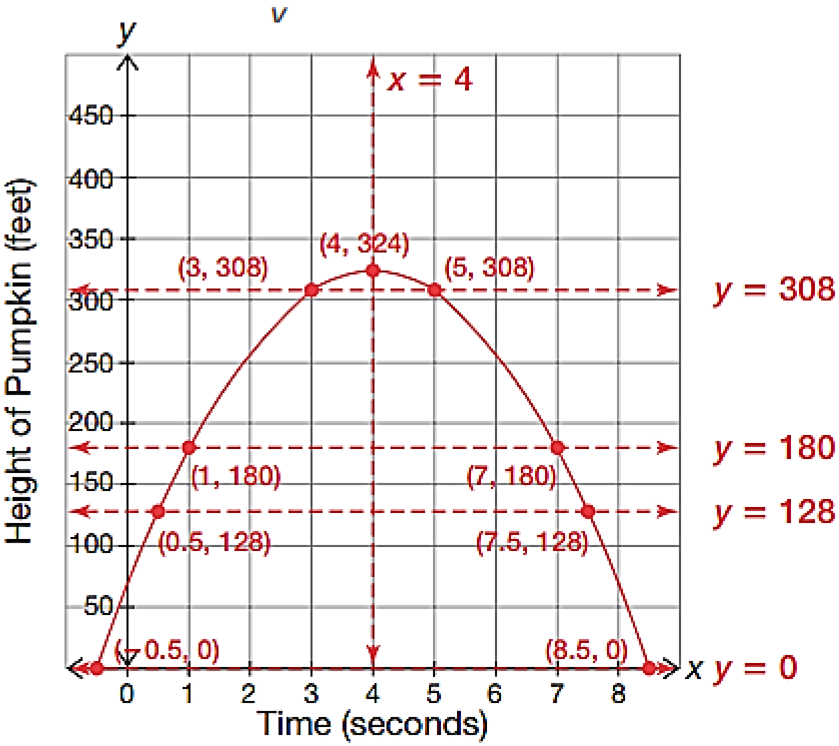
4. Use the information from Questions 2 and 3 to construct a graph.

a. Plot and label the vertex. Then draw and label the axis of symmetry.

b. Plot and label the points that correspond to the answers from Question 3.

c. Plot and label the points symmetric to the points from part (b).

d. Plot and label the zeros



5. Analyze the symmetric points.

a. What do you notice about the y-coordinates?

The y-coordinates are the same.

b. What do you notice about each point's horizontal distance from the axis of symmetry?

Each x-coordinate is the same distance from the axis of symmetry.

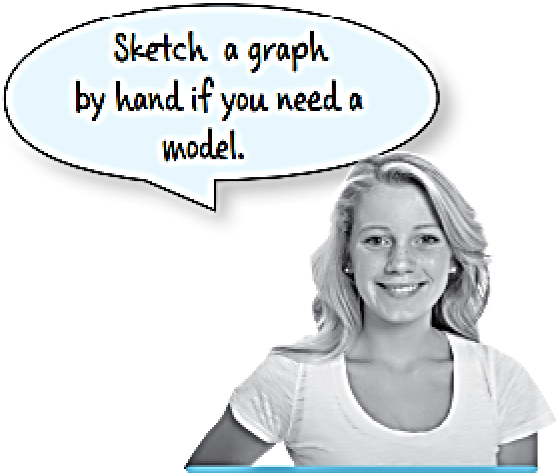
6. How does the x-coordinate of each symmetric point compare to the x-coordinate of the vertex .

The x-coordinate of the vertex is halfway between the x-coordinates of the symmetric points.

Talk the Talk

Use the given information to answer each question. Do not use a graphing calculator. Show your work.

1. Determine the axis of symmetry of each parabola.



a. The *x*-intercepts of the parabola are (1, 0) and (5, 0).

$$x = 3 \;\; because \;\; \frac{1+5}{2} = \frac{6}{2} = 3$$

b. The *x*-intercepts of the parabola are (−3.5, 0) and (4.1, 0).

$$x = 0.3 \;\; because \;\; \frac{-3.5+4.1}{2} = \frac{0.6}{2} = 0.3$$

c. Two symmetric points on the parabola are (−7, 2) and (0, 2).

$$x = -3.5 \;\; because \;\; \frac{-7+0}{2} = \frac{-7}{2} = -3.5$$

d. Describe how to determine the axis of symmetry given the *x*-intercepts of a parabola.

Add the x-coordinates together and divide by 2 to find the value of the axis of symmetry.

2. Determine the location of the vertex of each parabola.

a. The function $f(x) = x^2 + 4x + 3$ has the axis of symmetry $x = -2$.

The axis of symmetry is $x = -2$, so the x-coordinate of the vertex is -2.

The y-coordinate when $x = -2$ is:

$$f(-2) = (-2)^2 + 4(-2) + 3$$
$$= 4 - 8 + 3$$
$$= -1$$

The vertex is at $(-2, -1)$.

b. The equation of the parabola is $y = x^2 - 4$, and the x-intercepts are $(-2, 0)$ and $(2, 0)$.

The axis of symmetry is $x = 0$ because $\frac{-2 + 2}{2} = \frac{0}{2} = 0$. So, the x-coordinate of the vertex is 0.

The y-coordinate when $x = 0$ is:

$$f(0) = 0^2 - 4$$
$$= 0 - 4$$
$$= -4$$

The vertex is at $(0, -4)$.

c. The function $f(x) = x^2 + 6x - 5$ has two symmetric points $(-1, -10)$ and $(-5, -10)$.

The axis of symmetry is $x = -3$ because $\frac{-1 + (-5)}{2} = \frac{-6}{2} = -3$. So, the x-coordinate of the vertex is -3.

The y-coordinate when $x = -3$ is:

$$f(-3) = (-3)^2 + 6(-3) - 5$$
$$= 9 + (-18) - 5$$
$$= -14$$

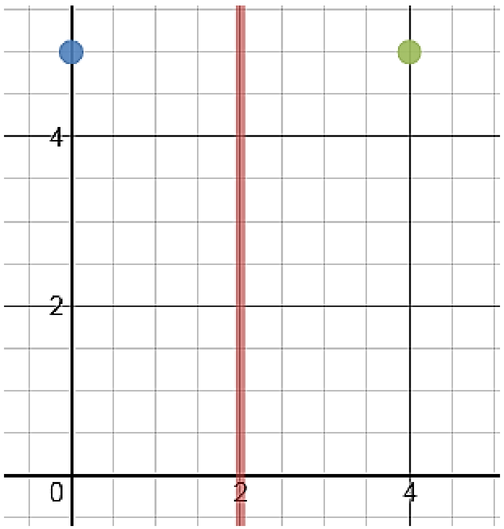
The vertex is at $(-3, -14)$.

d. Describe how to determine the vertex of a parabola given the equation and the axis of symmetry.

The axis of symmetry gives the x-coordinate of the vertex. Substitute this x-value into the equation for the parabola to determine the y-coordinate of the vertex.

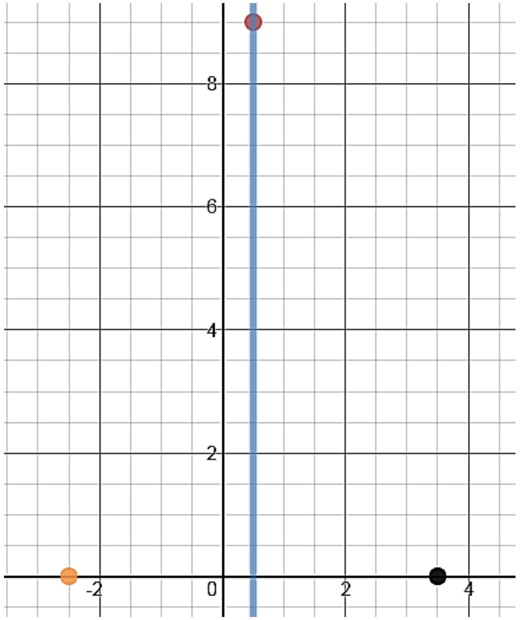
3. Determine another point on each parabola.

a. The axis of symmetry is x = 2.
A point on the parabola is (0, 5).
Another point on the parabola:



Another point on the parabola is (4, 5).

b. The vertex is (0.5, 9).
An x-intercept is (-2.5, 0).
Another point on the parabola:



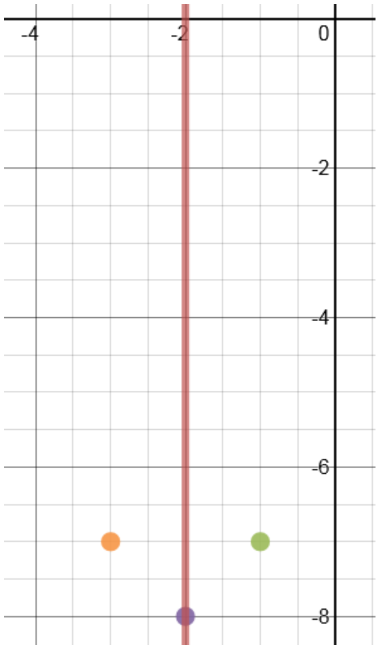
Another point on the parabola is the other *x*-intercept, (3.5, 0).

c. The vertex is (−2, −8).

A point on the parabola is (−1, −7).

Another point on the parabola:

Another point on the parabola is (−3, −7).



d. Describe how to determine another point on a parabola and the axis of symmetry.

The axis of symmetry is equidistant from the x-coordinates of 2 symmetric points. It is the vertical line that intersects the vertex.

Calculate the horizontal distance from the axis of symmetry to a point on the graph. Go the ***same*** distance in the opposite direction from the axis of symmetry to determine another symmetric point.