

11.6

The Form Is “Key”

Vertex Form of a Quadratic Function

LEARNING GOALS

In this lesson, you will:

- Determine key characteristics of parabolas using a graphing calculator.
- Determine key characteristics of parabolas given their equations in standard form.
- Determine key characteristics of parabolas given their equations in factored form.
- Determine key characteristics of parabolas given their equations in vertex form.
- Write equations of parabolas given key characteristics of their graphs.

KEY TERM

- vertex form

PROBLEM 1 Analyzing Forms

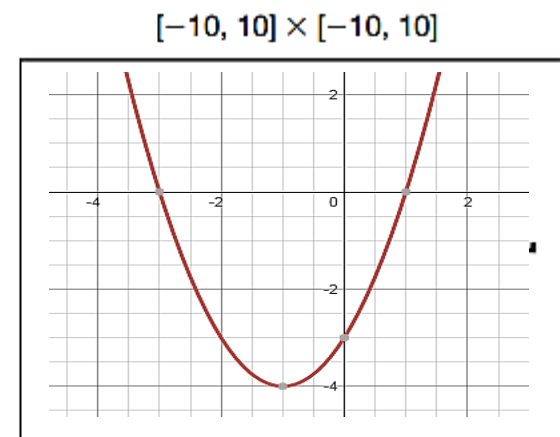
Use Desmos.com

- For each function:
 - complete the table of values.
 - sketch the graph in the space shown using a window with the given bounds.
 - tell whether the parabola opens up or down.
 - determine the location of the vertex.
 - determine the zero(s).
 - determine the y-intercept.

You have 20 minutes to complete problem 1, #'s 1-7. (Pages 662- 667)

a. $f(x) = x^2 + 2x - 3$

x	$f(x)$
-2	-3
-1	-4
0	-3
1	0
2	5



parabola opens: up

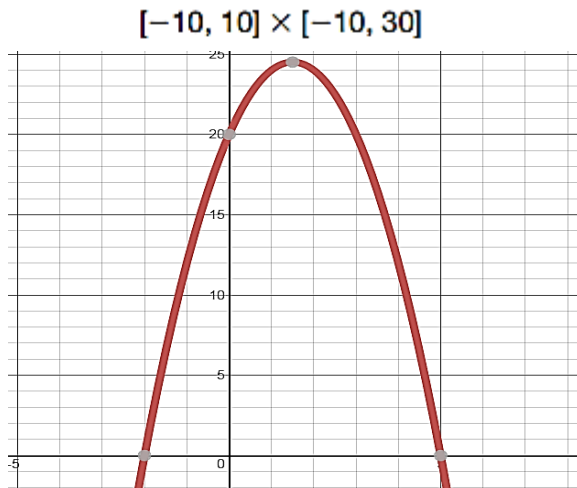
vertex: $(-1, -4)$

zero(s): $(-3, 0)$ and $(1, 0)$

y-intercept: $(0, -3)$

b. $f(x) = -2x^2 + 6x + 20$

x	$f(x)$
-2	0
-1	12
0	20
1	24
2	24



parabola opens: down

vertex: (1.5, 24.5)

zero(s): (-2, 0) and (5, 0)

y-intercept: (0, 20)

The quadratic functions in Question 1 are written in standard form, $f(x) = ax^2 + bx + c$.

2. In Question 1, you determined if the parabola opens up or down, the location of the vertex, the zeros, and the y-intercept using Desmos.

Which, if any of those key characteristics can you determine directly from a quadratic function when it is written in standard form?

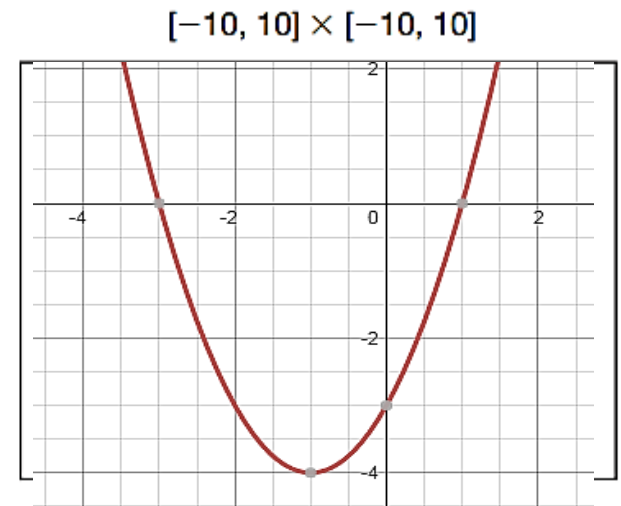
If a is +, the parabola opens UP. If a is -, the parabola opens DOWN.
 c = y-intercept

3. For each function:

- complete the table of values.
- sketch the graph in the space shown using a window with the given bounds.
- tell whether the parabola opens up or down.
- determine the location of the vertex.
- determine the zero(s).
- determine the y-intercept.

a. $f(x) = (x - 1)(x + 3)$

x	$f(x)$
-2	-3
-1	-4
0	-3
1	0
2	5



parabola opens: up

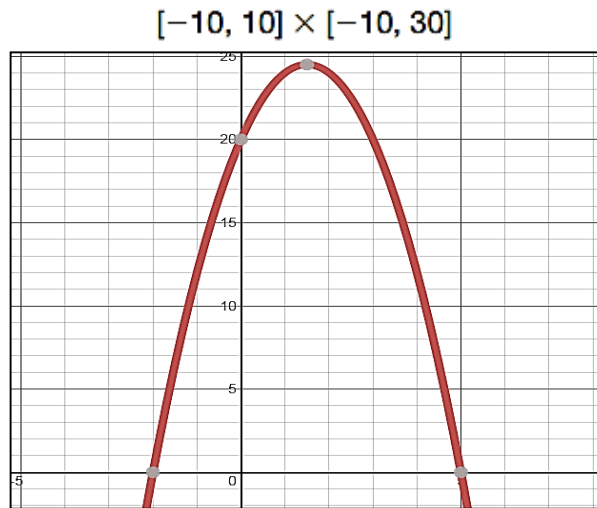
vertex: (-1, -4)

zero(s): (-3, 0) and (1, 0)

y-intercept: (0, -3)

b. $f(x) = -2(x + 2)(x - 5)$

x	$f(x)$
-2	0
-1	12
0	20
1	24
2	24



parabola opens: down

vertex: (1.5, 24.5)

zero(s): (-2, 0) and (5, 0)

y-intercept: (0, 20)

4. Compare your answers in Question 1 with your answers in Question 3.

What do you notice? **1a & 3a are the same functions. So are 1b & 3b. One is written in standard form, the other is factored form.**

The quadratic functions in Question 3 are written in factored form, $f(x) = a(x - r_1)(x - r_2)$.

You learned about the factored form of a quadratic function earlier in this chapter.

5. What key characteristics can you determine directly from a quadratic function when it is written in factored form?

The sign of a tells you if the parabola opens UP or DOWN.

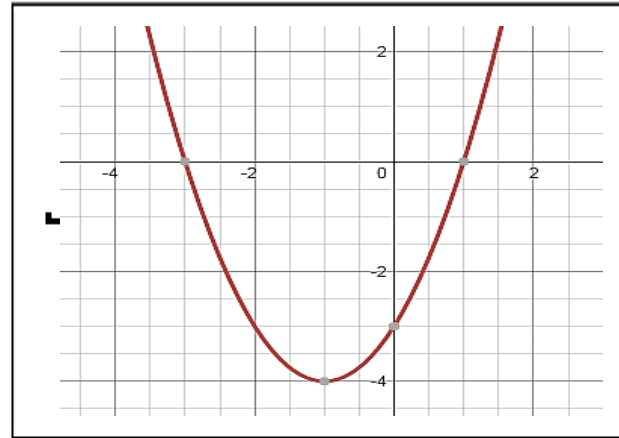
r_1 and $r_2 = x$ -intercepts. $(r_1, 0)$ and $(r_2, 0)$

6. Graphing, continued:

a. $f(x) = (x + 1)^2 - 4$

x	$f(x)$
-2	-3
-1	-4
0	-3
1	0
2	5

$[-10, 10] \times [-10, 10]$



parabola opens: up

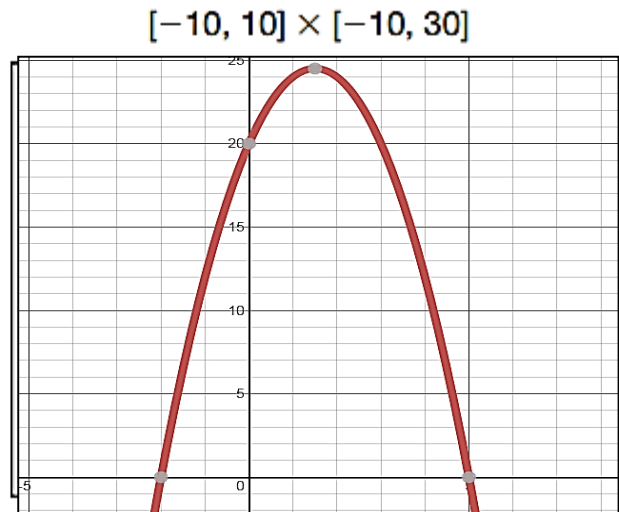
vertex: $(-1, -4)$

zero(s): $(-3, 0)$ and $(1, 0)$

y-intercept: $(0, -3)$

b. $f(x) = -2(x - 1.5)^2 + 24.5$

x	$f(x)$
-2	0
-1	12
0	20
1	24
2	24



parabola opens: down

vertex: (1.5, 24.5)

zero(s): (-2, 0) and (5, 0)

y-intercept: (0, 20)

7. What do you notice when you compare the functions in Question 6 with the functions in Question 1?

1a, 3a & 6a are the same functions. So are 1b, 3b & 6b. The functions are written in standard form, factored form & vertex form.

The quadratic functions in Question 6 are written in *vertex form*. A quadratic function written in **vertex form** is in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$.

8. What does the variable h represent in the vertex form of a quadratic function?

$h = x$ -coordinate of the vertex

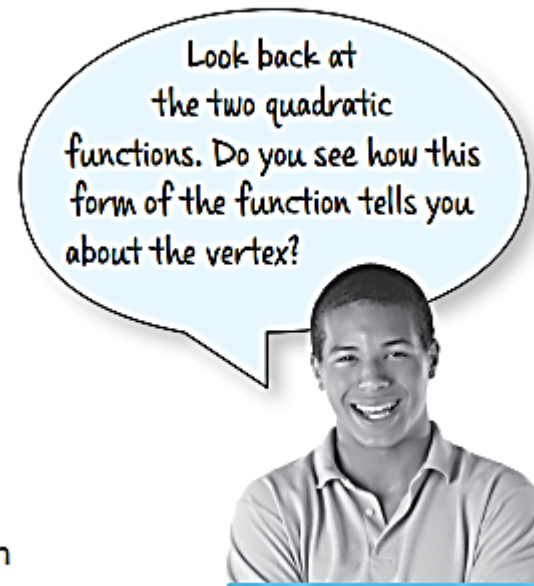
9. What does the variable k represent in the vertex form of a quadratic function?

$k = y$ -coordinate of the vertex



10. What key characteristics can you determine directly from the quadratic function when it is written in vertex form?

The sign of a tells you if the parabola opens UP or DOWN.
 $(h, k) =$ vertex



Skip to #2 on page 669.

2. Use Desmos.com to rewrite each quadratic function. First, determine the vertex of each and write the function in vertex form. Then, determine the zero(s) of each and write the function in factored form.

a. $h(x) = x^2 - 8x + 12$

vertex: (4, -4)

vertex form: $h(x) = (x - 4)^2 - 4$

zero(s): (6, 0) and (2, 0)

factored form: $h(x) = (x - 6)(x - 2)$

Vertex Form

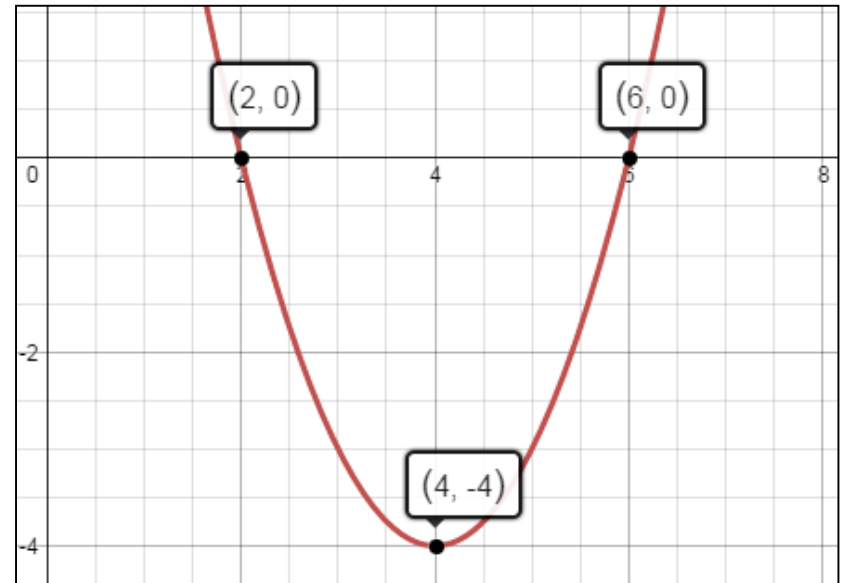
$$f(x) = a(x - h)^2 + k$$

Vertex: (h, k)

Factored Form

$$f(x) = a(x - r_1)(x - r_2)$$

zeros: (r₁, 0) & (r₂, 0)



2. Use Desmos.com to rewrite each quadratic function. First, determine the vertex of each and write the function in vertex form. Then, determine the zero(s) of each and write the function in factored form.

a. $h(x) = x^2 - 8x + 12$

vertex: (4, -4)

vertex form: $h(x) = (x - 4)^2 - 4$

zero(s): (6, 0) and (2, 0)

factored form: $h(x) = (x - 6)(x - 2)$

c. $w(x) = -x^2 - 4x$

vertex: (-2, 4)

vertex form: $w(x) = -(x + 2)^2 + 4$

zero(s): (0, 0) and (-4, 0)

factored form: $w(x) = -(x - 0)(x + 4)$ or $w(x) = -x(x + 4)$

b. $r(x) = -2x^2 + 6x + 20$

vertex: (1.5, 24.5)

vertex form: $r(x) = -2(x - 1.5)^2 + 24.5$

zero(s): (5, 0) and (-2, 0)

factored form: $r(x) = -2(x - 5)(x + 2)$

d. $c(x) = 3x^2 - 3$

vertex: (0, -3)

vertex form: $c(x) = 3(x - 0)^2 - 3$ or $c(x) = 3x^2 - 3$

zero(s): (-1, 0) and (1, 0)

factored form: $c(x) = 3(x + 1)(x - 1)$

3. Identify the form(s) of each quadratic function as either standard form, factored form, or vertex form. Then state all you know about each quadratic function's key characteristics, based only on the given equation of the function.

a. $f(x) = -(x - 1)^2 + 9$

vertex form

Parabola opens down

Vertex = (1, 9)

c. $f(x) = -\frac{1}{2}(x - 3)(x + 2)$

factored form

Parabola opens down

X-intercepts = (3, 0) and (-2, 0)

b. $f(x) = x^2 + 4x$

standard form

Parabola opens up

Y-intercept = (0, 0)

d. $f(x) = x^2 - 5$

vertex form and standard form

Parabola opens up

Y-intercept = (0, -5)

Vertex = (0, -5)

4. Use the given information to write a possible equation for each quadratic function.

a. The zeros are -4 and 6, and the parabola opens down.

$$f(x) = -a(x + 4)(x - 6)$$

b. The vertex is (0, 3), and the parabola opens up.

$$f(x) = a(x - 0)^2 + 3 = ax^2 + 3$$

c. The vertex is (-1, 1), and the parabola opens down.

$$f(x) = -a(x + 1)^2 + 1$$

d. The zeros are 0 and 2, and the parabola opens up.

$$f(x) = a(x - 0)(x - 2) = ax(x - 2) = ax^2 - 2ax$$

5. Complete the graphic organizers on the next two pages using the standard form of the function given. Check the appropriate boxes for each function form.

Standard Form

Equation: $f(x) = x^2 + 2x - 3$

Identify the key features from standard form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

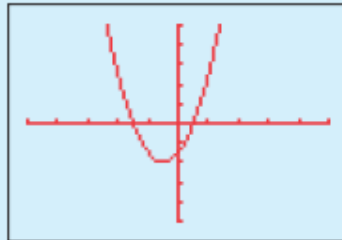
Factored Form

Equation: $f(x) = (x - 1)(x + 3)$

Identify the key features from factored form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

Graph of the Quadratic Function



Vertex Form

Equation: $f(x) = (x + 1)^2 - 4$

Identify the key features from vertex form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

Standard Form

Equation: $f(x) = -2x^2 + 6x + 20$

Identify the key features from standard form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

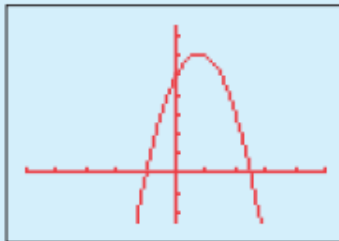
Factored Form

Equation: $f(x) = -2(x + 2)(x - 5)$

Identify the key features from factored form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept

Graph of the Quadratic Function



Vertex Form

Equation: $f(x) = -2(x - 1.5)^2 + 24.5$

Identify the key features from vertex form:

- parabola opens up/down
- location of vertex
- zeros
- y-intercept