



Learning Goals:

- To identify and factor the difference of two squares.
- To identify and factor perfect square trinomials.
- To find the solutions to quadratic equations by factoring.

Notes (Page 752)

1. Multiply the binomials.

a. $(x-4)(x+4) = x^2 - 16$

Binomial or Trinomial?

$(x+4)(x+4) = x^2 + 8x + 16$

Binomial or Trinomial?

$(x-4)(x-4) = x^2 - 8x + 16$

Binomial or Trinomial?

b. $(3x-1)(3x+1) = 9x^2 - 1$

Binomial or Trinomial?

$(3x+1)(3x+1) = 9x^2 + 6x + 1$

Binomial or Trinomial?

$(3x-1)(3x-1) = 9x^2 - 6x + 1$

Binomial or Trinomial?

2. What patterns do you see? Hint: Look at the "+" and "-" signs?

The only difference in the 1st set of binomials is a sign change. The 2nd set and the 3rd set of binomials are the same. The middle term of the polynomial has the same sign as the set of binomials. There is no middle term in the 1st set because the inner + outer terms cancel each other out.

3. Multiply these binomials. Do you recognize the pattern?

$(ax-b)(ax+b) = a^2x^2 - b^2$

Difference of Two Squares

$(ax+b)(ax+b) \text{ or } (ax+b)^2 = a^2x^2 + 2abx + b^2$

Perfect Square Trinomial (+)

$(ax-b)(ax-b) \text{ or } (ax-b)^2 = a^2x^2 - 2abx + b^2$

Perfect Square Trinomial (-)

Save these formulas!

4. Group the quadratic equations in #1 into two categories: the Difference of Two Squares or Perfect Square Trinomials.

Difference of Two Squares

$(x+4)(x-4) = x^2 - 16$

$(3x+1)(3x-1) = 9x^2 - 1$

Perfect Square Trinomials

$(x+4)(x+4) = x^2 + 8x + 16$

$(x-4)(x-4) = x^2 - 8x + 16$

$(3x+1)(3x+1) = 9x^2 + 6x + 1$

$(3x-1)(3x-1) = 9x^2 - 6x + 1$

5. Factor each difference of two squares.

a. $x^2 - 4 = \frac{(x+2)(x-2)}{\sqrt{\quad} \sqrt{\quad}}$
 b. $4x^2 - 9 = \frac{(2x+3)(2x-3)}{\sqrt{\quad} \sqrt{\quad}}$
 c. $x^4 - 16 = \frac{(x^2+4)(x^2-4)}{\sqrt{\quad} \sqrt{\quad}}$

Some expressions can be factored even further! If so, keep factoring!

$(x^4 - 16) = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x+2)(x-2)$
 Factor $\xrightarrow{\quad}$

$(x^4 - y^4) = (x^2 + y^2)(x^2 - y^2) = \frac{(x^2 + y^2)(x+y)(x-y)}{\text{Factor } \xrightarrow{\quad}}$

6. Try to factor $x^2 + 49$. What do you get?

It cannot be factored, because there is a "+" sign.

7. Factor each perfect square trinomial.

$x^2 + 10x + 25 = \frac{(x+5)(x+5)}{\sqrt{\quad} \sqrt{\quad}}$ or $\frac{(x+5)^2}{\sqrt{\quad} \sqrt{\quad}}$

$x^2 - 24x + 144 = \frac{(x-12)(x-12)}{\sqrt{\quad} \sqrt{\quad}}$ or $\frac{(x-12)^2}{\sqrt{\quad} \sqrt{\quad}}$

$4x^2 + 20x + 25 = \frac{(2x+5)(2x+5)}{\sqrt{\quad} \sqrt{\quad}}$ or $\frac{(2x+5)^2}{\sqrt{\quad} \sqrt{\quad}}$

$36x^2 - 36x + 9 = \frac{9(4x^2 - 4x + 1)}{\sqrt{\quad} \sqrt{\quad}} = \frac{9(2x-1)(2x-1)}{\sqrt{\quad} \sqrt{\quad}}$

Factor out the GCF!

~~$16x^4 - 1 = \frac{(4x^2+1)(4x^2-1)}{\sqrt{\quad} \sqrt{\quad}} = \frac{(4x^2+1)(2x+1)(2x-1)}{\sqrt{\quad} \sqrt{\quad}}$~~
 Factor

8. Calculate the roots of each quadratic equation.

a. $x^2 - 12x + 36 = 0$

$(x-6)(x-6) = 0$

$x - 6 = 0$

$x = 6$

b. $9x^2 - 25 = 0$

$(3x+5)(3x-5) = 0$

$3x + 5 = 0$ or $3x - 5 = 0$

$3x = -5$

$3x = 5$

$x = \frac{-5}{3}$

$x = \frac{5}{3}$

9. Calculate the zeros of each function.

a. $f(x) = 25x^2 + 20x + 4$

$$(5x + 2)(5x + 2) = 0$$

$$5x + 2 = 0$$

$$5x = -2$$

$$x = -\frac{2}{5}$$

b. $f(x) = 9x^2 + 1$

Cannot be factored,

There are no real zeros.

c. $f(x) = 9 - 24x + 16x^2 \rightarrow 16x^2 - 24x + 9$ (standard form)

$$(4x - 3)(4x - 3) = 0$$

$$4x - 3 = 0$$

$$4x = 3$$

$$x = \frac{3}{4}$$

d. $f(x) = \frac{1}{4}x^2 - 1$

$$\left(\frac{1}{2}x + 1\right)\left(\frac{1}{2}x - 1\right) = 0$$

$$\frac{1}{2}x + 1 = 0 \quad \text{or} \quad \frac{1}{2}x - 1 = 0$$

$$\frac{1}{2}x = -1$$

$$x = -2$$

$$\frac{1}{2}x = 1$$

$$x = 2$$