Scouting for Prizes!
Modeling Linear Inequalities

2.3

LEARNING GOALS

In this lesson, you will:

- Write and solve inequalities.
- Analyze a graph on a coordinate plane to solve problems involving inequalities.
- Interpret how a negative rate affects how to solve an inequality.

KEY TERM

- solve an inequality

PROBLEM 1 Popcorn Pays Off

Alan's camping troop is selling popcorn to earn money for an upcoming camping trip. Each camper starts with a credit of $25 toward his sales, and each box of popcorn sells for $3.75.

Alan can also earn bonus prizes depending on how much popcorn he sells. The table shows the different prizes for each of the different sales levels. Each troop member can choose any one of the prizes at or below the sales level.

Sales (dollars)	Gift Cards (2 of each value)	Bonus Prizes
$250	$10	
$350	$15	
$450	$20	
$600		Cyclone Sprayer
$650	$30	
$850	$40	
$1100	$55	
$1300	$75	
$1500		Choose your prize!
$1800	$110	
$2300	$150	
$2500		6% toward college scholarship

1. Write a function, $f(b)$, to show Alan's total sales as a function of the number of boxes of popcorn he sells.

   $f(b) = 3.75b + 25$

2. Analyze the function you wrote.

   a. Identify the independent and dependent quantities and their units.

      Independent Quantity (IQ) = # of boxes sold
      Dependent Quantity (DQ) = total sales (in dollars)

   b. What is the rate of change and what does it represent in this problem situation?

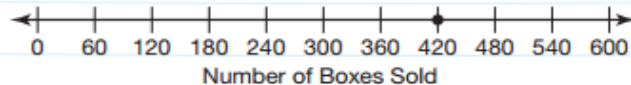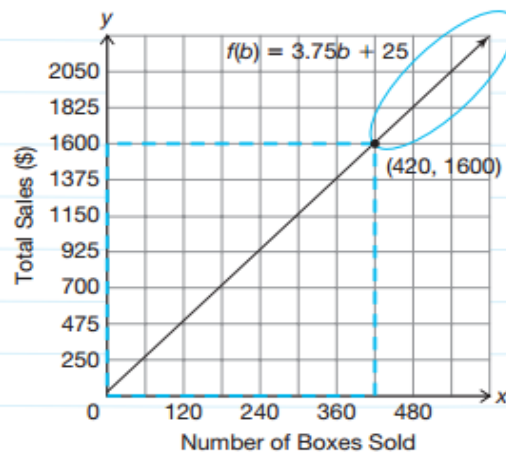   **Rate of Change** = $3.75.   It represents the cost of each box of popcorn.

   c. What is the y-intercept and what does it represent in this problem situation?

   **y-intercept** = 25. This means that every troop member starts with a $25 credit toward the total sales.

The graph shown represents the change in the total sales as a function of boxes sold. The oval and box represent the total sales at specific intervals.


$$f(b) = 3.75b + 25$$

(420, 1600)

Total Sales ($) vs *Number of Boxes Sold*

Number line: 0 60 120 180 240 300 360 420 480 540 600
Number of Boxes Sold

> The point at (420, 1600) means that at 420 boxes sold, the total sales is equal to $1600. This is represented on the number line as a closed point at 420. When f(b) = 1600, then b = 420.

> The box represents all the numbers of boxes sold, b, that would earn Alan $1600 or less. When f(b) ≤ 1600 then b ≤ 420.

Number line: 0 60 120 180 240 300 360 420 480 540 600
Number of Boxes Sold
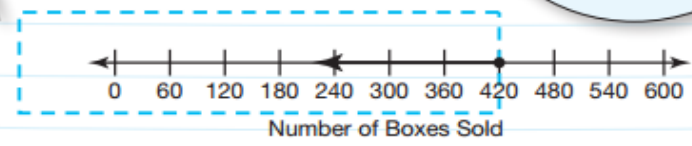
> The oval represents all the numbers of boxes sold, b, that would earn Alan more than $1600. When f(b) > 1600, then b > 420.

Number line: 0 60 120 180 240 300 360 420 480 540 600
Number of Boxes Sold
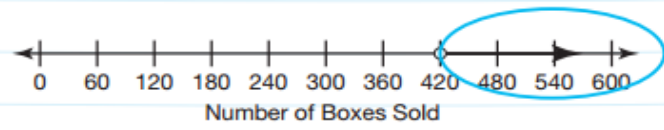
3. Explain the difference between the open and closed circles on the number lines.

Open Circle = **O** = the point IS NOT included in the solution
Closed Circle = ● = the point IS included in the solution

4. Use the graph to answer each question. Write an equation or inequality statement for each.

a. How many boxes would Alan have to sell to earn at least $925?

Alan would have to sell **at least** 240 boxes, $b \geq 240$.

b. How many boxes would Alan have to sell to earn less than $2050?

Alan would have to sell **fewer than** 540 boxes, $b < 540$.

c. How many boxes would Alan have to sell to earn exactly $700?

Alan would have to sell **exactly** 180 boxes, $b = 180$.

How does determining the intersection point help you determine your answers?

What's Your Strategy—Your Algebraic Strategy?

Another way to determine the solution set of an inequality is to solve it algebraically. To **solve an inequality** means to determine the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: You want to isolate the variable on one side of the inequality symbol.

In order to earn two $55 gift cards, Alan's total sales, $f(b)$, needs to be at least $1100. You can set up an inequality and solve it to determine the number of boxes Alan needs to sell.

$$f(b) \geq 1100$$

$$3.75b + 25 \geq 1100$$

Solve the inequality in the same way you would solve an equation.

$$3.75b + 25 \geq 1100$$

$$3.75b + 25 - 25 \geq 1100 - 25$$

$$3.75b \geq 1075$$

$$\frac{3.75b}{3.75} \geq \frac{1075}{3.75}$$

$$b \geq 286.66 \ldots$$

Alan needs to sell at least 287 boxes of popcorn to earn two $55 gift cards.

1. Why was the answer rounded to 287?

You have to sell a "whole number" of boxes, so you round up.

2. Write and solve an inequality for each. Show your work.

   a. What is the greatest number of boxes Alan could sell and still not have enough to earn the Cyclone Sprayer?

   $3.75b + 25 < 600$

   $3.75b < 575$

   $b < 153.\overline{3}$

   Alan can sell at most 153 boxes of popcorn and still not have enough to earn the Cyclone Sprayer.

   b. At least how many boxes would Alan have to sell to be able to choose his own prize?

   $3.75b + 25 \geq 1500$

   $3.75b \geq 1475$

   $b \geq 393.\overline{3}$

   Alan would need to sell at least 394 boxes to choose his own prize.