5.5

Radical! Because It's Cliché!

Properties of Rational Exponents

LEARNING GOALS

In this lesson, you will:

- Simplify expressions with negative exponents.
- Simplify expressions with rational exponents.
- Write negative powers as positive powers.
- Write rational powers using radicals.
- Find the *n*th root of a number.
- Write an expression in radical form.

KEY TERMS

- cube root
- index
- *n*th root
- radicand
- rational exponent

You can say that a number b is a **cube root** of a number a if $b^3 = a$.

So, 5 is a cube root of 125 because $5^3 = 125$.

A cube root is indicated by the symbol $\sqrt[3]{\ }$. The number 3 is called the **index** of the radical. So, $\sqrt[3]{125} = 5$.

5. Complete each statement.

a. $\sqrt[3]{8} =$ 2

Because $2^3 = 8$

b. $\sqrt[3]{-27} =$ −3

Because $(-3)^3 = -27$

You can extend the idea of square roots and cube roots to solve equations. If n represents a positive number, then a number b is the ***n*th root** of a if $b^n = a$.

For example, 2 is the 4th root of 16 because $2^4 = 16$.

6. Complete each statement.

a. The number 3 is the 5th root of 243 because 3^5 = 243.

b. The number −2 is the cube root of −8 because $(-2)^3 =$ −8.

c. The number 4 is the 6th root of 4096 because $4^6 = 4096$.

The *n*th root of a number *a* is designated as $\sqrt[n]{a}$, where *n* is the index of the radical and *a* is the radicand, which is the value within the radical.

7. Complete each statement. The first one has been done for you.

a. $\sqrt{100} =$ 10 because $10^2 = 100$.

b. $\sqrt[3]{216} =$ 6 because $6^3 = 216$.

c. $\sqrt[4]{81} =$ 3 because $3^4 = 81$.

d. $\sqrt[5]{-32} =$ -2 because $(-2)^5 = -32$.

8. Notice that a power can be positive or negative, depending on the base and the exponent.

a. When the exponent of a power is an even number, and the base is a positive number, is the value of the power positive or negative? How do you know?

The value is POSITIVE because the product of positive numbers is always positive. $5^2 = 25$

b. When the exponent of a power is an even number, and the base is negative, is the value of the power positive or negative? How do you know?

The value is POSITIVE because the product of an even number of negative #s is always positive.

$(-5)^2 = 25$

c. When the exponent of a power is an odd number, and the base is a positive number, is the value of the power positive or negative? How do you know?

The value is POSITIVE because the product of positive numbers is always positive. $2^3 = 8$

d. When the exponent of a power is an odd number, and the base is negative, is the value of the power positive or negative? How do you know?

The value is NEGATIVE because the product of an odd number of negative #s is always negative.

$(-2)^3 = -8$

PROBLEM 3 Boat Speed

The hull speed of a boat depends on the length of the hull at the waterline. An equation that relates the speed s in knots and the length, r in feet of the hull at the waterline is $s = 1.34\sqrt{r}$.

1. What is the hull speed of a boat that has a length of 16 feet at the waterline? Show your work.

$s = 1.34\sqrt{16}$

$s = 1.34(4)$ The speed of the boat is 5.36 knots.

$s = 5.36$

2. Write an equation that you can use to determine the length of a boat at its waterline if the boat's hull speed is 6.7 knots.

$6.7 = 1.34\sqrt{r}$

3. Transform the equation you wrote to isolate the radical on one side of the equation. Show your work.

$6.7 = 1.34\sqrt{r}$

$5 = \sqrt{r}$

4. What is the length of the boat at the waterline? How do you know?

The length of the boat at the waterline is 25 feet because the square root of 25 is 5.

Skip problem #5.

A **rational exponent** is an exponent that is a rational number. You can write each *n*th root using a rational exponent. If *n* is an integer greater than 1, then $\sqrt[n]{a} = a^{\frac{1}{n}}$.

6. Write each radical as a power.

a. $\sqrt[3]{7}$ $7^{\frac{1}{3}}$ **b.** $\sqrt[5]{x}$ $x^{\frac{1}{5}}$ **c.** $\sqrt{y}$ $y^{\frac{1}{2}}$

7. Write each power as a radical.

a. $8^{\frac{1}{4}}$ $\sqrt[4]{8}$ **b.** $z^{\frac{1}{5}}$ $\sqrt[5]{z}$ **c.** $m^{\frac{1}{3}}$ $\sqrt[3]{m}$

8. Write the equation for the hull speed of a boat using a rational exponent.

Original: $s = 1.34\sqrt{r}$

$s = 1.34r^{\frac{1}{2}}$

Go to problem #10.

10. Write each expression in radical form. Show your work and simplify your answer, if possible.

a. $4^{\frac{3}{2}}$ $\sqrt{4^3} = \sqrt{64} = 8$

or $\left(\sqrt{4}\right)^3 = 2^3 = 8$

b. $5^{\frac{3}{4}}$ $\sqrt[4]{5^3} = \sqrt[4]{125}$

c. $x^{\frac{4}{5}}$ $\sqrt[5]{x^4}$

d. $y^{\frac{2}{3}}$ $\sqrt[3]{y^2}$

11. Write each expression in rational exponent form. Show your work and simplify your answer, if possible.

a. $(\sqrt[4]{2})^3$ $2^{\frac{3}{4}}$

b. $(\sqrt{5})^4$ $5^{\frac{4}{2}} = 5^2 = 25$

c. $(\sqrt[5]{x})^8$ $x^{\frac{8}{5}}$

d. $(\sqrt[5]{y})^{10}$ $y^{\frac{10}{5}} = y^2$