7.2

Working the System

Systems of Linear Inequalities

LEARNING GOALS

In this lesson, you will:

- Write and graph systems of linear inequalities.
- Determine solutions to systems of linear inequalities.
- Algebraically prove solutions and non-solutions of systems of linear inequalities.
- Graph systems of linear inequalities using a graphing calculator.

KEY TERMS

- constraints
- solution of a system of linear inequalities

PROBLEM 1 Whitewater Rafting

Chase is an experienced whitewater rafter who guides groups of adults and children out on the water for amazing adventures. The super-raft he uses can hold 800 pounds of weight. Any weight greater than 800 pounds will cause the raft to sink, hit more rocks, and maneuver more slowly.

1. Chase estimates the weight of each adult as approximately 200 pounds and the weight of each child under age sixteen as approximately 100 pounds. Chase charges adults $75 and children under age sixteen $50 to ride down the river with him. His goal is to earn at least $150 each rafting trip.
   a. Write an inequality to represent the most weight Chase can carry in terms of rafters. Define your variables.

      x = # of adult rafters

      y = # of child rafters

      $200x + 100y \leq 800$

   b. Write an inequality to represent the least amount of money Chase wants to collect for each rafting trip.

      $75(x - 1) + 50y \geq 150$

      Chase rides for free.

   c. Write a system of linear inequalities to represent the maximum weight of the raft and the minimum amount of money Chase wants to earn per trip.

$$\begin{cases} 200x + 100y \leq 800 \\ 75(x-1) + 50y \geq 150 \end{cases}$$

In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region on the graph.

2. Let's consider the past two trips that Chase guides. Determine whether each combination of rafters is a solution of the system of linear inequalities. Then describe the meaning of the solution in terms of this problem situation.

a. First Trip: Chase guides 2 adults and 2 children.

Don't forget to count Chase as an adult.

$$200x + 100y \le 800$$
$$200(3) + 100(2) \le 800$$
$$600 + 200 \le 800$$
$$800 \le 800$$

$$75(x-1) + 50y \ge 150$$
$$75(2) + 50(2) \ge 150$$
$$150 + 100 \ge 150$$
$$250 \ge 150$$

Yes. This is a solution because both inequalities are TRUE.

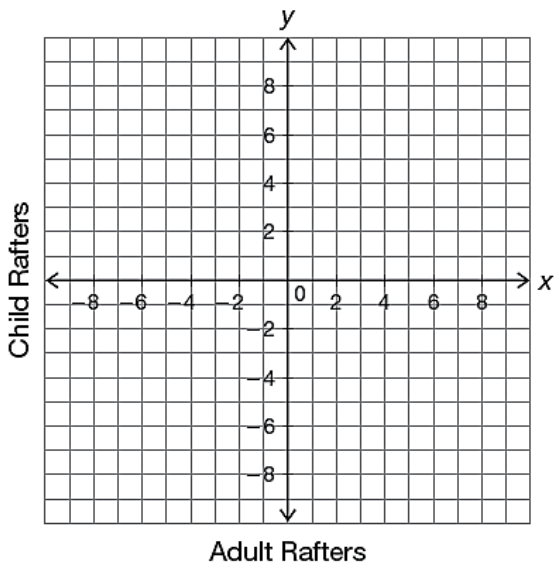
b. Second Trip: Chase guides 5 adults.

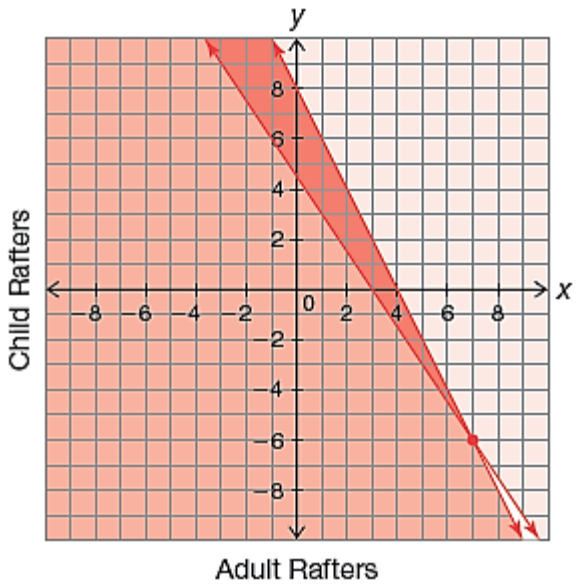
$$200x + 100y \le 800$$
$$200(6) + 100(0) \le 800$$
$$1200 \not\le 800$$

$$75(x-1) + 50y \ge 150$$
$$75(6-1) + 50(0) \ge 150$$
$$75(5) \ge 150$$
$$375 \ge 150$$

No. This is NOT a solution because one of the inequalities is NOT TRUE.

3. Graph the system of linear inequalities on the coordinate plane shown.





The **solution of a system of linear inequalities** is the intersection of the solutions to each inequality. Every point in the intersection region satisfies the solution.

4. Analyze your graph.

   a. Describe the possible number of solutions for a system of linear inequalities.

Each linear inequality is represented as a half-plane. As long as the half-planes overlap, the system of inequalities can have infinitely many solutions.

   b. Is the intersection point a solution to this system of inequalities? Why or why not?

Yes, it is a solution, but it does not make sense. The intersection point (7, -6) mean you have a “negative” number of children in the raft.

c. Identify three different solutions of the system of linear inequalities you graphed. What do the solutions represent in terms of the problem situation?

Answers will vary.
Three possible solutions are (3, 2), (2, 4), and (4, 0).

The solution (3, 2) means that Chase plus 2 other adults and 2 children weigh at most 800 pounds and pay at least $150.

d. Determine one combination of adults and children that is *not* a solution for this system of linear inequalities. Explain your reasoning.

Answers will vary.

The point (2, 1) does not represent a solution.

$200(2) + 100(1) \leq 800$

$400 + 100 \leq 800$

$500 \leq 800$ ✓

$75(2 - 1) + 50(1) \geq 150$

$75 + 50 \geq 150$

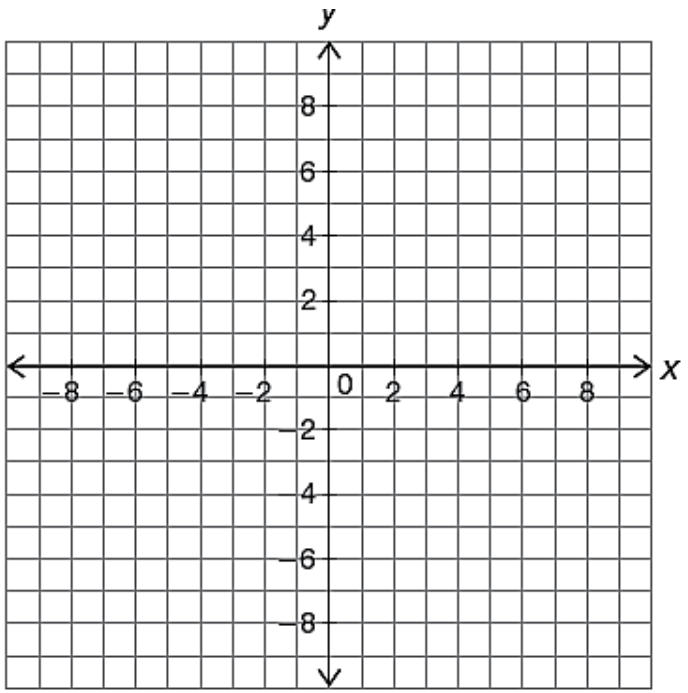
$125 \geq 150$ ✗

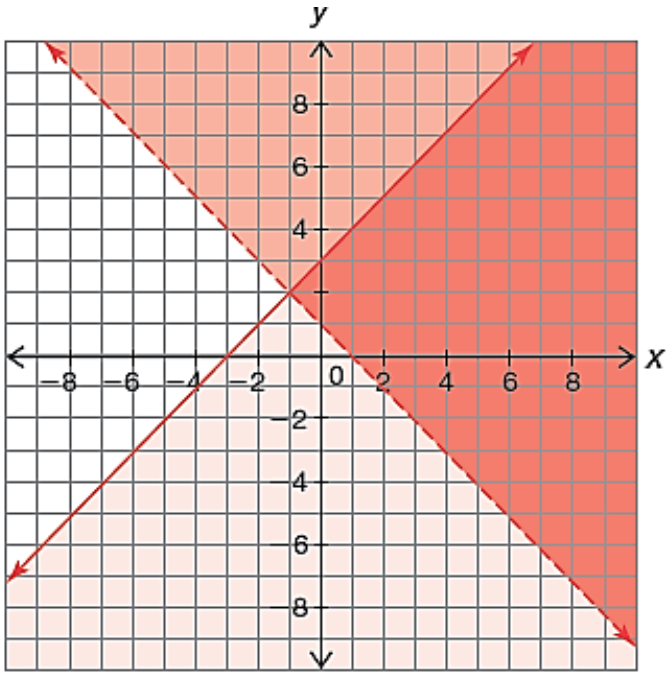
Although Chase, 1 other adult, and 1 child are within the weight limit for the raft, the money earned is less than $150. Because this ordered pair does not produce true statements in both inequalities, it is not a solution.

5. Analyze the solution set of the system of linear inequalities shown.

$$\begin{cases} x + y > 1 \\ -x + y \leq 3 \end{cases}$$

a. Graph the system of linear inequalities.





Name two points that are solutions to the system of inequalities.
(Answers will vary).

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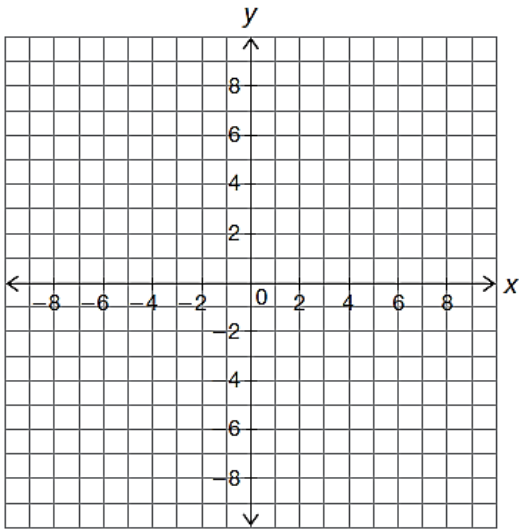
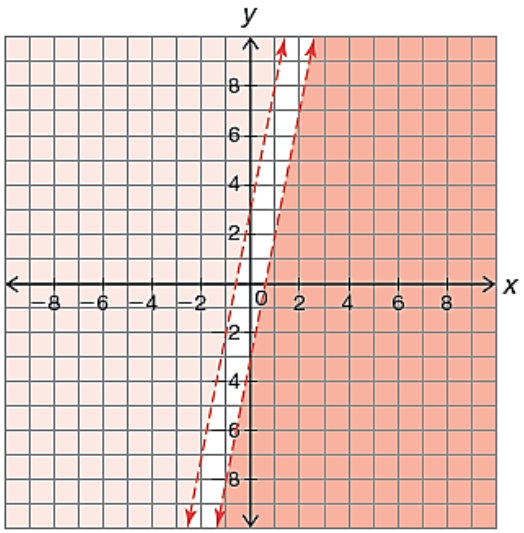
6. Solve each system of linear inequalities by graphing the solution set. Then identify two points that are solutions of the system.

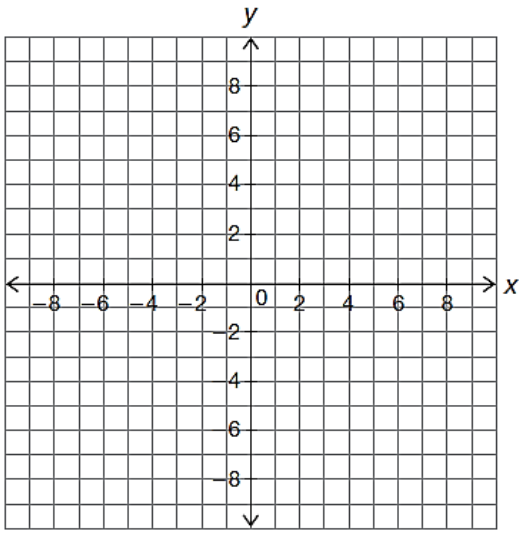
a. $\begin{cases} y > 5x + 3 \\ y < 5x - 3 \end{cases}$

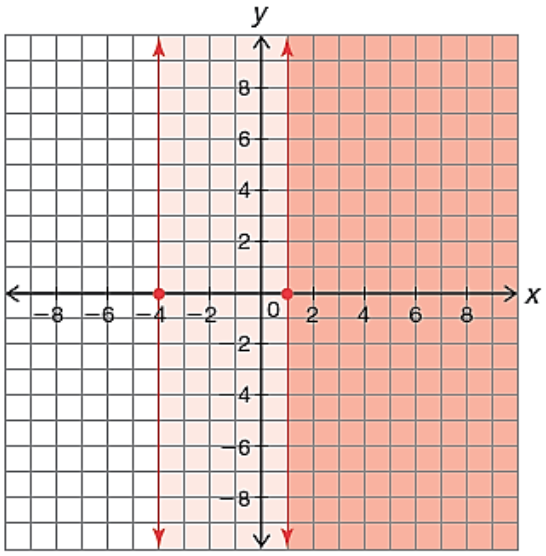




The shaded regions or half-planes do NOT overlap so there are NO SOLUTIONS.

b. $\begin{cases} x \geq -4 \\ x \geq 1 \end{cases}$





Two possible solutions are (2, 0) and (5, 5).