11.2

Just U and I

Comparing Linear and Quadratic Functions

LEARNING GOALS

In this lesson, you will:

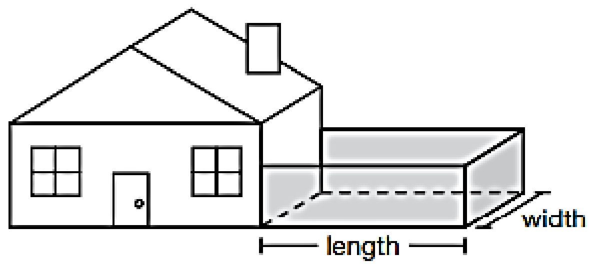
- Identify linear and quadratic functions from multiple representations.
- Compare graphs, tables, and equations for linear and quadratic functions.
- Analyze graphs of linear and quadratic functions.
- Determine if a function is linear or quadratic by analyzing the first and second differences.

KEY TERMS

- leading coefficient
- second differences

PROBLEM 1 Deciding on the Dimensions

Two dog owners have 16 yards of fencing to build a dog run beside their house. The dog owners want the run to be in the shape of a rectangle, and they want to use the side of their house as one side of the dog run. A rough sketch of what they have in mind is shown.



1. Complete the table to show different widths, lengths, and areas that can occur with sixteen yards of fencing.

Remember, area = length x width

Width	Length	Area
yards	yards	square yards
0	8	0
2	7	14
4	6	24
6	5	30
8	4	32
10	3	30
12	2	24
14	1	14
16	0	0

2. Describe what happens to the ***length*** **as the width** of the dog run **increases** . Why do you think this happens?
The length decreases as the width increases.

3. Describe what happens to the ***area*** **as the width** of the dog run **increases.**
The area increases, then decreases.

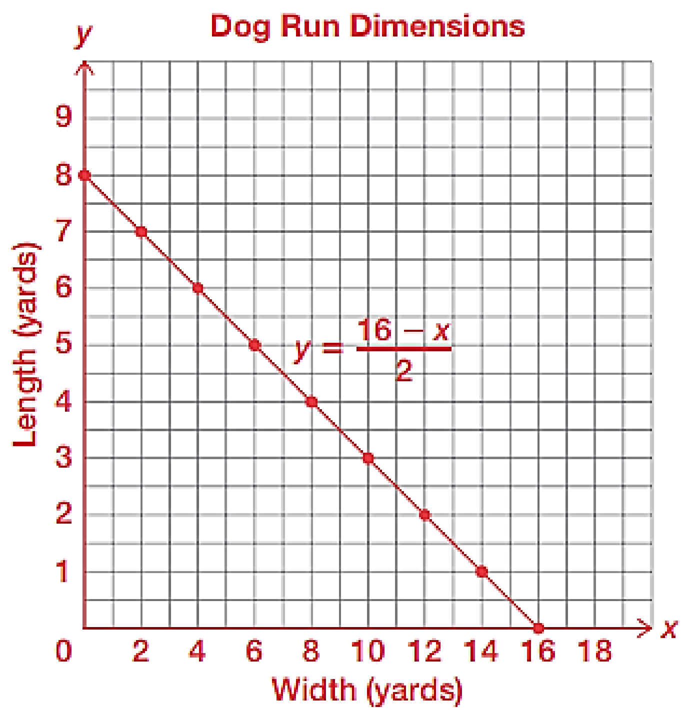
4. Describe what happens to the ***length and area*** **as the width** of the dog run **decreases**.
The length increases as the width decrease. The area increases, then decreases.

5. Describe what happens to the ***width and area*** **as the length** of the dog run **increases**. Describe what happens to the ***width and area*** **as the length** of the dog run **decreases**.
As the length increases, the width decreases. The area increases, then decreases.
As the length decreases, the width increases. The area increases, then decreases.

6. Compare how the area changes as the width changes to how the area changes as the length changes.
The area increases, then decreases when either measurement increases or decreases.

7. Let $L(w)$ represent the length of the dog run as a function of the width. Create a graph to show this relationship. First, choose your bounds and intervals. Be sure to label your graph clearly.

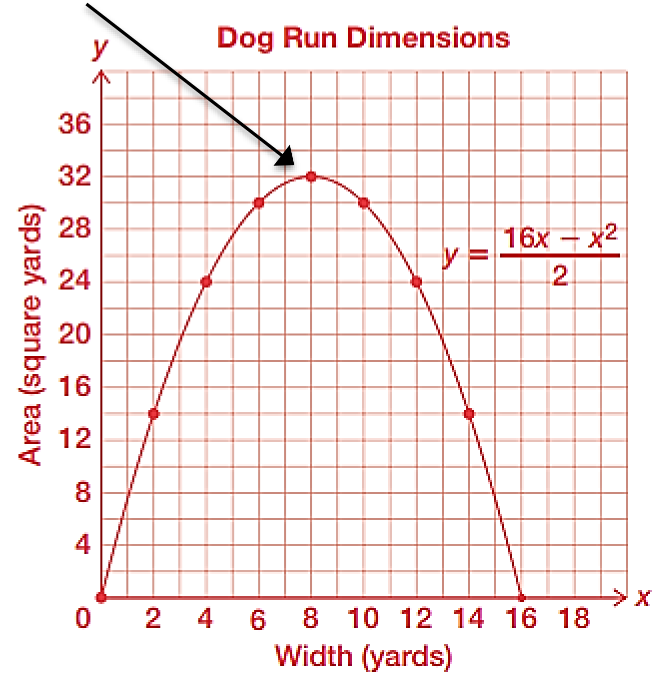
Variable Quantity	Lower Bound	Upper Bound	Interval
Width	0	20	2
Length	0	10	1



8. Let $A(w)$ represent the area of the dog run as a function of the width. Create a graph to show this relationship. First, choose your bounds and intervals. Be sure to label your axes and name your graph.

Variable Quantity	Lower Bound	Upper Bound	Interval
Width	0	20	2
Area	0	40	4

High point = absolute maximum



9. Let's compare and contrast the graphs of the two functions. L(w): The length of the dog run as a function of the width. A(w): The area of the dog run as a function of the width .

a. Describe the type of function represented by each graph. Explain your reasoning.

Graph of L(w) = a straight line so the function is linear.
Graph of A(w) = a parabola so the function is a quadratic.

b. State the domain in terms of each function and the problem situation.

Domain of L(w) = the set of all real numbers. For the problem situation, the domain is $0 < w < 16$, where w is the width.
Domain of A(w) = the set of all real numbers. For the problem situation, the domain is $0 < w < 16$, where w is the width.

c. Determine the y-intercepts of each graph and interpret the meaning of each in terms of the problem situation.

Y-intercept of L(w) = 8. It represents the length when the width is 0.
Y-intercept of A(w) = 0. It represents the area when the width is 0.
It does not make sense to have a dog run with a width of zero.

d. Describe the rates of change for each graph.

L(w), or the linear function, has a constant negative rate of change.
A(w), or the quadratic function, has a varying rate of change.

10. Determine the dimensions that provide the greatest area. Use the graphical representations to explain your reasoning.

The greatest area is 32 square yards, the width = 8 yards and length = 4 yards.

Look at the graph of the parabola. The coordinates of the absolute maximum (8, 32) give you width and greatest area.

Use the graph of the linear function to find the corresponding length. When the width is 8, the length is 4.