PROBLEM 2 U and I

Tables A and B represent two different functions. One is a linear function, and one is a quadratic function.

Table A

x	$A(x)$
−2	7.5
−1	7.25
0	7
1	6.75
2	6.5

+1
+1
+1
+1

Table B

x	$B(x)$
−2	−15
−1	−7.25
0	0
1	6.75
2	13

+1
+1
+1
+1

1. Which table do you think represents each type of function? Explain your reasoning.
   Table A represents a ***linear function.***
   The difference in the values for $A(x)$ is ***always the same,*** -0.25.
   Table B represents a ***quadratic function.***
   The difference in the values for $B(x)$ ***changes***.

2. Calculate the first differences for each function. What patterns do you notice?
   For Table A, the 1st differences are the ***same***. For Table B, the 1st differences ***change***.

The graphs of the two functions are shown. The two equations that represent the linear and quadratic graphs are:

$$y = -\frac{1}{4}x + 7$$

$$y = -\frac{1}{4}x^2 + 7x$$

Table A

$$A(x) = -\frac{1}{4}x + 7$$

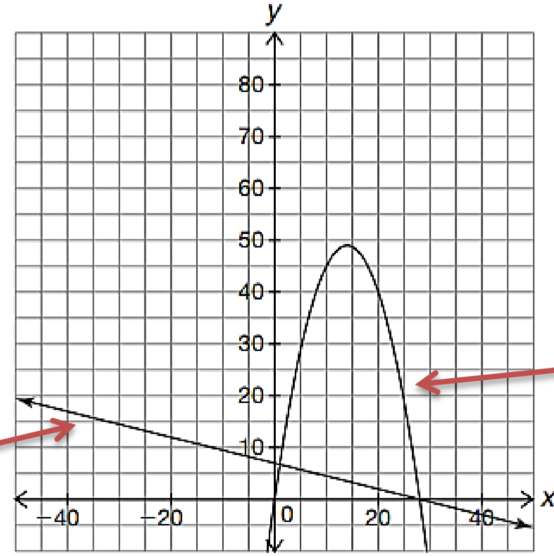


Table B

$$B(x) = -\frac{1}{4}x^2 + 7x$$

3. Identify the graph that represents Table A and the graph that represents Table B. Then rewrite each equation as the function A(x) or B(x) and label the graph appropriately. Was your prediction in Question 1 correct? See the graph.

4. Describe the rate of change for each graph. Explain your reasoning.

The linear function has a ***constant*** negative rate of change because the line is decreasing.

The quadratic function has a ***varying*** rate of change. The graph increases, then decreases because it opens downwards.

5. Determine the y-intercept of each function. Explain how you know.

Y-intercept for the linear function is at (0, 7).

Y-intercept for the quadratic function is at (0, 0).

The y-intercept is where the graph crosses the y-axis so x = 0. It is also where x = 0 in the table.

The **leading coefficient** of a function is the numerical coefficient of the term with the greatest power. Recall that a power has two elements: the base and the exponent.

6. Identify the leading coefficient of each function. Then, describe how the sign of the leading coefficient affects the behavior of each graph.
For the linear function, the leading coefficient = -¼.
Since, it is negative, the linear function is decreasing.
For the quadratic function, the leading coefficient = -¼.
Since it is negative, the quadratic function opens downward.

Let's explore the table of values one step further and analyze the second differences. **Second differences** are the differences between consecutive values of the first differences.

7. Calculate the second differences for each function. What do you notice?
For the linear function, 2nd differences are ***always*** 0.
For the quadratic function, 2nd differences are ***always*** -0.5. ***They stay the same***!

PROBLEM 3 Second Differences

1. Analyze the form of each equation and determine if it is linear or quadratic. Then complete each table to calculate the first and second differences.

a. $y = 2x$ *Linear*

x	y
−3	-6
−2	-4
−1	-2
0	0
1	2
2	4
3	6

First Differences
2
2
2
2
2
2

Second Differences
0
0
0
0
0

b. $y = 2x^2$ *Quadratic*

x	y
−3	18
−2	8
−1	2
0	0
1	2
2	8
3	18

First Differences
-10
-6
-2
2
6
10

Second Differences
4
4
4
4
4

c. $y = -x + 4$ Linear

x	y	First Differences	Second Differences
−3	7		
		-1	
−2	6		0
		-1	
−1	5		0
		-1	
0	4		0
		-1	
1	3		0
		-1	
2	2		0
		-1	
3	1		

d. $y = -x^2 + 4$ Quadratic

x	y	First Differences	Second Differences
−3	-5		
		5	
−2	0		-2
		3	
−1	3		-2
		1	
0	4		-2
		-1	
1	3		-2
		-3	
2	0		-2
		-5	
3	-5		

2. What do you notice about the first and second differences of the:

a. linear functions.

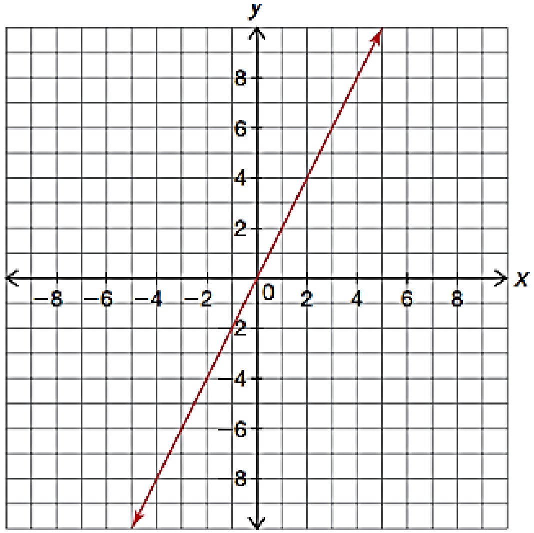
Linear functions have ***constant*** 1st differences and 2nd differences of 0.

b. quadratic functions.

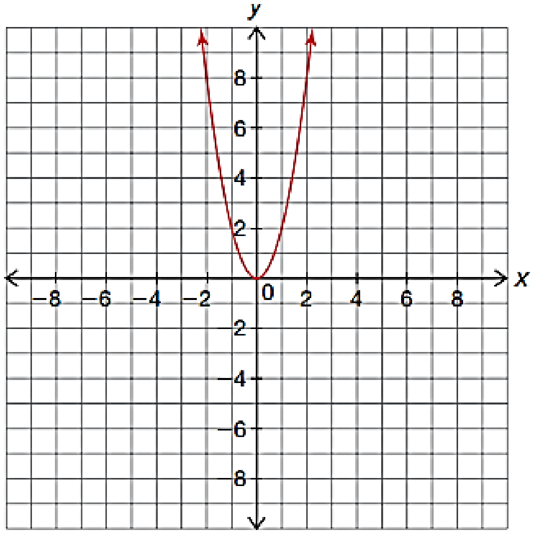
Quadratic functions have ***changing*** 1st differences and ***constant*** 2nd differences.

3. Sketch the graphs represented by the equations in Question 1.

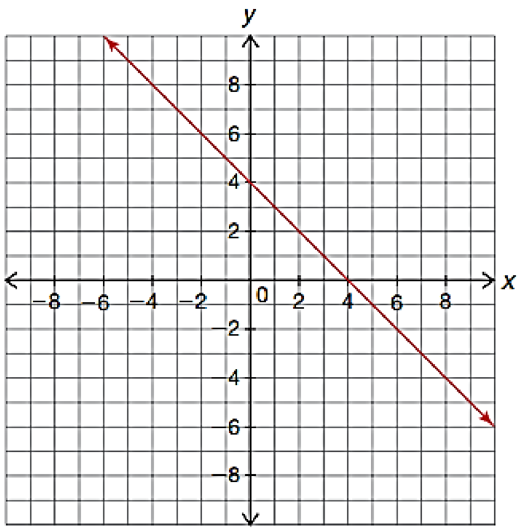
a. $y = 2x$



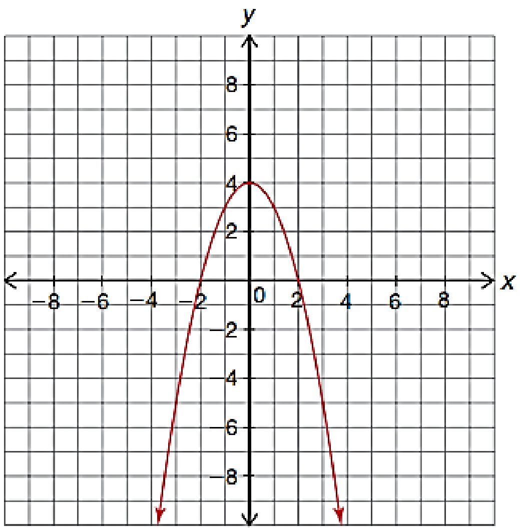
b. $y = 2x^2$



c. $y = -x + 4$



d. $y = -x^2 + 4$



4. Compare the signs of the first and second differences for each function and its graph.

a. How do the signs of the ***first differences for a linear function*** relate to the graph either increasing or decreasing?

1st differences are positive (+) so the linear function is increasing (↑) .

1st differences are negative (–) so the linear function is decreasing (↓) .

b. How do the signs of the ***first differences and the signs of the second differences for quadratic functions*** relate to the graph of the quadratic either increasing or decreasing or opening upward or downward?

1st differences are positive (+) so the quadratic function is increasing(↑) .

1st differences are negative (–) so the quadratic function is decreasing (↓) .

2nd differences are positive (+) so the parabola opens upward ().

2nd differences are negative (–) so the parabola opens downward ().

Talk the Talk

1. Describe how to determine when an equation represents a:

   a. linear function.

   $f(x) = mx + b$

   b. quadratic function .

   $f(x) = ax^2 + bx + c \quad where \ x \neq 0$

2. Describe how to determine when a table of values represents a:

   a. linear function.

   Linear functions have ***constant 1st differences*** and 2nd differences of 0.

   b. quadratic function

   Quadratic functions have either increasing or decreasing 1st differences and ***constant 2nd differences***.