Warm Up

Determine whether each quadratic function contains an absolute minimum or an absolute maximum.

1. $f(x) = -\frac{1}{2}x^2 + 3x - 1$

Leading coefficient is NEGATIVE → Parabola opens DOWNward → Absolute MAXIMUM

2. $f(x) = x^2 - 3x + 1$

Leading coefficient is POSITIVE → Parabola opens UPward → Absolute MINIMUM

3. $f(x) = -5x(2 - x)$

$f(x) = -10x + 5x^2$

Leading coefficient is POSITIVE → Parabola opens UPward → Absolute MINIMUM

4. $f(x) = 2x(1 - x)$

$f(x) = 2x - 2x^2$

Leading coefficient is NEGATIVE → Parabola opens DOWNward → Absolute MAXIMUM

11.3

Walking the . . . Curve?

Domain, Range, Zeros, and Intercepts

LEARNING GOALS

In this lesson, you will:

- Describe the domain and range of quadratic functions.
- Determine the x-intercept(s) of a graph of a quadratic function.
- Understand the relationship of the zeros of a quadratic function and the x-intercepts of its graph.
- Analyze quadratic functions to determine intervals of increase and decrease.
- Solve a quadratic function graphically.

KEY TERMS

- vertical motion model
- zeros
- interval
- open interval
- closed interval
- half-closed interval
- half-open interval

PROBLEM 1 Model Rocket

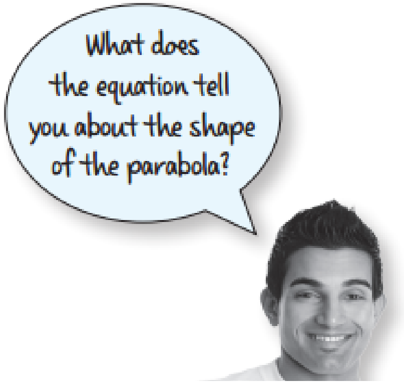
Suppose you launch a model rocket from the ground. You can model the motion of the rocket using a *vertical motion model*. A **vertical motion model** is a quadratic equation that models the height of an object at a given time. The equation is of the form

$$g(t) = -16t^2 + v_0 t + h_0,$$

where $g(t)$ represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial vertical velocity (speed) of the object in feet per second, and h_0 represents the initial height of the object in feet.

1. Why do you think it makes sense that this situation is modeled by a quadratic function?

   When a rocket is launched, its height increases over time until it reaches a maximum height, then it falls back to the earth. Its motion looks like a parabola.



Suppose the model rocket has an initial velocity of 160 feet per second.

2. Write a function, $g(t)$, to describe the height of the model rocket in terms of time t.

   $v_0 = 160$ feet per second; $h_0 = 0$ seconds

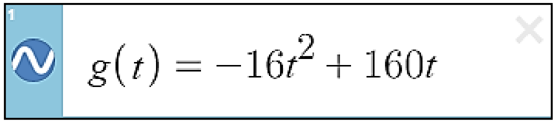
   $g(t) = -16t^2 + 160t$

3. Describe the independent and dependent quantities.

   Independent quantity (IQ) = time (in seconds)

   Dependent quantity (DQ) = height (in feet)

4. Use DESMOS to graph the function. Sketch the graph and label the axes.



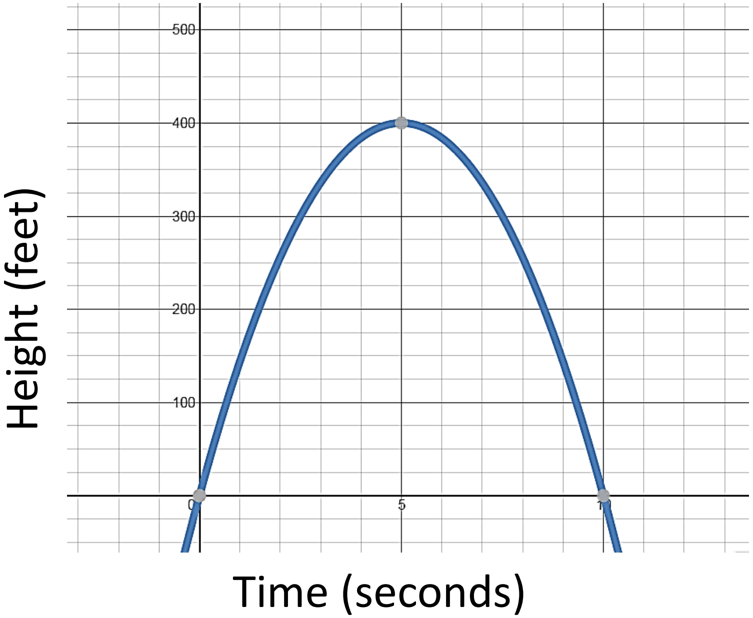
$g(t) = -16t^2 + 160t$

X-Axis

$-5 \le x \le 15$ Step: 5

Y-Axis

$-100 \le y \le 700$ Step: 100



5. Use DESMOS to answer each question.

a. What is the height of the model rocket at 6 seconds?

384 feet

b. After approximately how many seconds is the model rocket at a height of 200 feet?

At about 1.5 seconds and again at about 8.5 seconds

c. What is the maximum height of the model rocket? When is the rocket at its maximum height?

Hint: Find the vertex. The rocket reaches a maximum height of 400 feet after 5 seconds.

6. You can use DESMOS and intersection points to determine the x-intercepts of a quadratic function.

a. What linear function can you graph along with the quadratic function to determine the x-intercepts? Explain your reasoning.

The x-intercepts are the value(s) of x when the parabola crosses the x-axis. At these points, y = 0.

b. Determine the x-intercepts of $g(t)$. Then, interpret the meaning in terms of this problem situation.

x-intercepts: (0, 0) and (10, 0)

The x-intercepts indicate the time when the rocket is launched and the time it lands.

The x-intercepts of a graph of a quadratic function are also called the **zeros** of the quadratic function.

They are the x-values that make the function equal to zero.

7. Identify and describe the domain of the function in terms of the:

a. mathematical function you graphed.

Domain of the graph: All Real Numbers (ARN)

b. contextual situation.

Domain of this problem: $0 \leq x \leq 10$ or $0 \leq t \leq 10$

The model rocket is launched at 0 seconds and lands after 10 seconds. The other times on the x-axis don't make sense.

8. Identify and describe the range of the function in terms of the:

a. mathematical function you graphed.

Range of the graph: $y \leq 400$ or $g(t) \leq 400$

b. contextual situation.

Range of this problem: $0 \leq y \leq 400$ or $0 \leq g(t) \leq 400$

Negative y-values don't make sense because the rocket cannot have negative height.

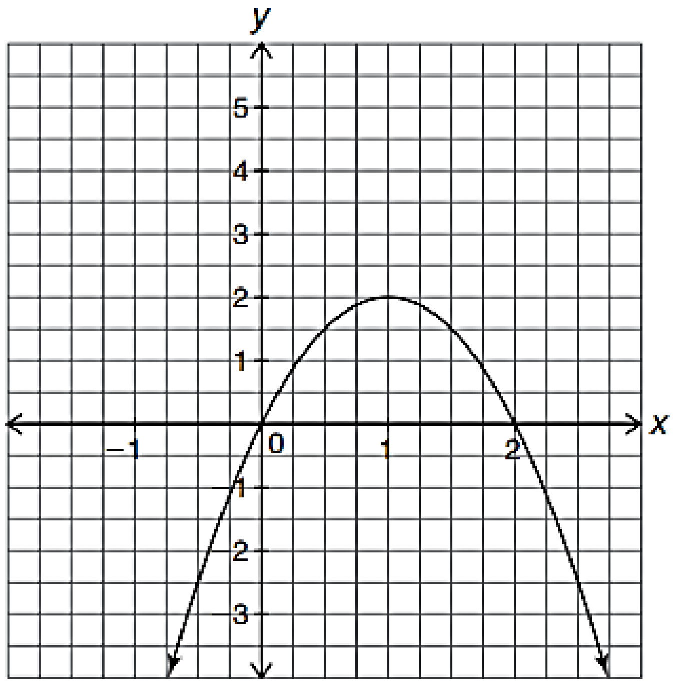
9. How is the range of a quadratic function related to its absolute maximum or minimum?

Absolute MAX or MIN limits the range because y-values do not exist above the maximum or below the minimum.

PROBLEM 2 Intervals of Increase and Decrease

For each function shown, identify the domain, range, zeros, and the intervals of increase and decrease.

1. The graph shown represents the function $f(x) = -2x^2 + 4x$.



Domain: All Real Numbers (ARN)

Range: $y \leq 2$ or $f(x) \leq 2$

y-intercept: (0, 0)

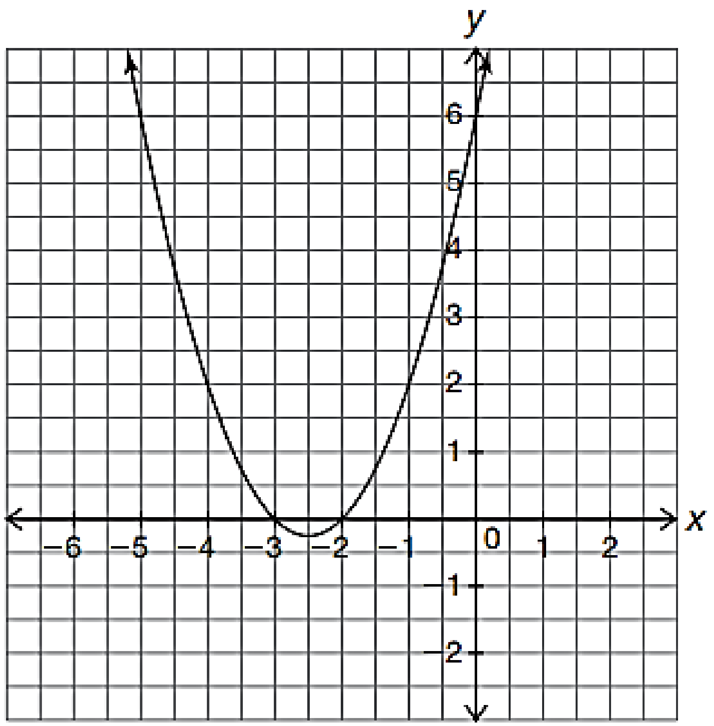
Zeros: (0, 0) and (2, 0)

~~Interval of increase:~~

[✓ Max] or Min?

~~Interval of decrease:~~

2. The graph shown represents the function $f(x) = x^2 + 5x + 6$.



Domain: All Real Numbers (ARN)

Range: $y \geq ¼$ or $f(x) \geq ¼$

y-intercept: (0, 6)

Zeros: (-3, 0) and (-2, 0)

~~Interval of decrease:~~

Max or [✓ Min?]

~~Interval of increase:~~