Warm Up

Use the Distributive Property to simplify each expression.

1. $f(x) = -x(x + 10)$ $\quad f(x) = -x^2 - 10x$

2. $f(x) = -8x(11 - x)$ $\quad f(x) = 8x^2 - 88x$

3. $f(x) = -6(x - 4)$ $\quad f(x) = -6x + 24$

4. $f(x) = 7(x + 1)$ $\quad f(x) = 7x + 7$

Write your answers in standard form: $f(x) = ax^2 + bx + c$, where $x \neq 0$

11.4

Are You Afraid of Ghosts?

Factored Form of a Quadratic Function

LEARNING GOALS

In this lesson, you will:

- Factor the greatest common factor from an expression.
- Write a quadratic function in factored form.
- Determine the *x*-intercepts from a quadratic function written in factored form.
- Determine an equation of a quadratic function given its *x*-intercepts.

KEY TERMS

- factor an expression
- factored form

Consider the expression $12x + 42$.

The greatest common factor of $12x$ and 42 is 6. Therefore, you can use the Distributive Property in reverse to rewrite the expression.

$$12x + 42 = 6(2x) + 6(7)$$
$$= 6(2x + 7)$$

So, the factored expression is $6(2x + 7)$.

The leading coefficient should be positive!

Practice Factoring:

1) $3x + 9$

GCF = 3

$3(x + 3)$

2) $4x - 20$

GCF = 4

$4(x - 5)$

3) $-5x - 25$

GCF = -5

$-5(x + 5)$

4) $-x + 8$

GCF = -1

$-1(x - 8)$

Skip to Page 647

A quadratic function written in **factored form** is in the form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$.

We are going to explore quadratic functions written in factored form….

$$f(x) = a(x - r_1)(x - r_2)$$

Open your chromebooks to DESMOS and turn to Page 649 in your book.

You and your partner will graph and compare each function in the blue boxes.

PROBLEM 2 Exploring Factored Form

1. A group of students are working together on the problem shown.

Write a quadratic function in factored form to represent a parabola that opens downward and has zeros at (4, 0) and (−1, 0).

Maureen

My function is

$k(x) = -(x - 4)(x + 1)$.

Micheal

My function is

$d(x) = \frac{1}{2}(x - 4)(x + 1)$.

Tim

My function is

$m(x) = 2(x - 4)(x + 1)$.

Tom

My function is

$k(x) = -2(x - 4)(x + 1)$.

Dianne

My function is

$t(x) = -0.5(x - 4)(x + 1)$.

Judy

My function is

$f(x) = -(x + 4)(x - 1)$.

a. Use your graphing calculator to graph each student's function. What are the similarities among all the graphs? What are the differences among the graphs.

All the graphs are parabolas.
Maureen, Tom, Dianne & Judy's graphs open downward.
Micheal & Tim's graphs open upward.
Tom & Tim's graphs are the same narrow width. The other graphs are wider.

b. How is it possible to have more than one correct function?

Their graphs meet the given characteristics. The parabolas open downwards and the x-intercepts are (4, 0) and (-1, 0).

c. What would you tell Micheal, Tim, and Judy to correct their functions.

Micheal & Tim's graphs open upward so their $\boldsymbol{a}$ needs to be negative.
Judy's zeros are (-4, 0) and (1, 0). She needs to change the sign of $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$.

d. How many possible functions can you write to represent the given characteristics? Explain your reasoning.

An infinite number of functions because the leading coefficient can be any negative number when the equation is written in factored form.

2. For a quadratic function written in factored form $f(x) = a(x - r_1)(x - r_2)$:

a. what does the sign of the variable a tell you about the graph?

The sign of a tells you whether the parabola opens upward (positive a) or downward (negative a).

b. what do the variables r_1 and r_2 tell you about the graph?

The variables r_1 and r_2 represent the x-coordinates of the x-intercepts. The x-intercepts are $(r_1, 0)$ and $(r_2, 0)$.

3. Use the given information to write a quadratic function in factored form, $f(x) = a(x - r_1)(x - r_2)$.

Values for *a* will vary.

a. The parabola opens upward and the zeros are (2, 0) and (4, 0).

$f(x) = a(x - 2)(x - 4)$ for $a > 0$ $\quad f(x) = 2(x-2)(x-4)$

b. The parabola opens downward and the zeros are (−3, 0) and (1, 0).

$f(x) = a(x + 3)(x - 1)$ for $a < 0$ $\quad f(x) = -6(x+3)(x-1)$

c. The parabola opens downward and the zeros are (0, 0) and (5, 0).

$f(x) = ax(x - 5)$ for $a < 0$ $\quad f(x) = -2x(x-5)$

d. The parabola opens upward and the zeros are (−2.5, 0) and (4.3, 0).

$f(x) = a(x + 2.5)(x - 4.3)$ for $a > 0$ $\quad f(x) = (x+2.5)(x-4.3)$

4. Compare your quadratic functions with your classmates' functions. How does the ***a*** value affect the shape of the graph?

When ***a*** < 1, the graph is wider.

The closer that ***a*** gets to 0, the wider the graph.

When ***a*** > 1, the graph becomes narrow.

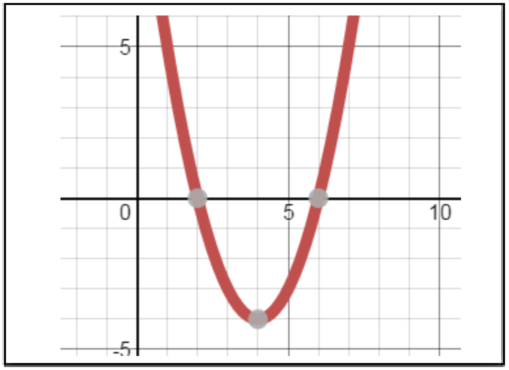
5. Use a graphing calculator to determine the zeros of each function. Sketch each graph using the zeros and y-intercept. Then, write the equation of the function in factored form.

Remember to factor out the GCF and keep the leading coefficient positive!

a. $h(x) = x^2 - 8x + 12$

zeros: $(6,0)$ *and* $(2,0)$

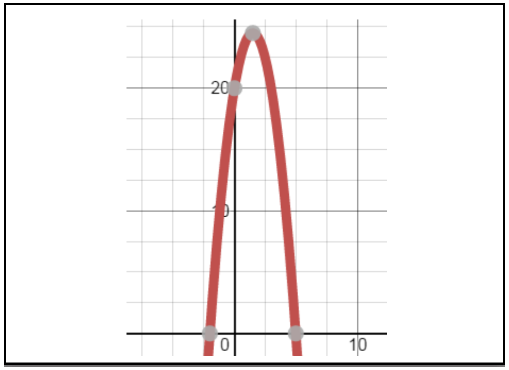
factored form: $h(x) = (x-6)(x-2)$



b. $r(x) = -2x^2 + 6x + 20$

zeros: $(-2,0)$ *and* $(5,0)$

factored form: $r(x) = -2(x+2)(x-5)$

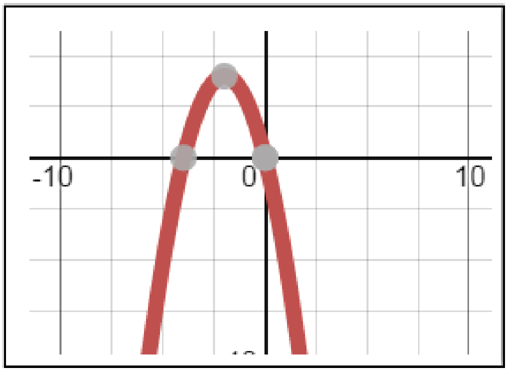


c. $w(x) = -x^2 - 4x$

zeros: $(-4,0)$ *and* $(0,0)$

factored form: $w(x) = -(x+4)(x-0)$

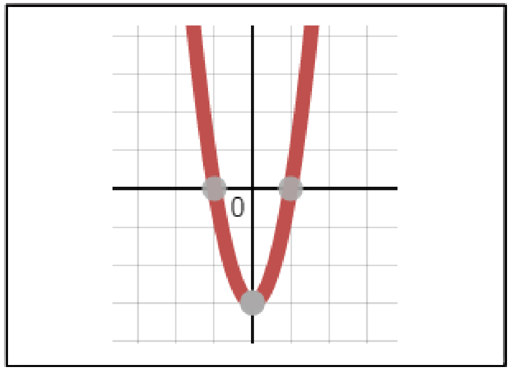
or $w(x) = -x(x+4)$



d. $c(x) = 3x^2 - 3$

zeros: $(-1,0)$ *and* $(1,0)$

factored form: $c(x) = 3(x+1)(x-1)$



6. Determine the zeros of the function. Write the function in factored form if it is not already in factored form.

a. $f(x) = (x - 2)(x - 7)$

The zeros are (2, 0) and (7, 0).

b. $v(x) = x(2x + 6)$

The zeros are (0, 0) and (-3, 0).

c. $g(x) = (x + 1)(5 - x)$

The zeros are (-1, 0) and (5, 0).

d. $p(x) = (-9 - 3x)(x + 4)$

The zeros are (-3, 0) and (-4, 0).