PROBLEM 2 Analyzing Equations and Graphs

1. Complete the table shown for the problem situation described in Problem 1, *Analyzing Tables*. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

output value	input value	rate of change

		What It Means	
Expression	**Unit**	**Contextual Meaning**	**Mathematical Meaning**
t	minutes	the time, in minutes, that the plane has been in the air	input value
1800	$\frac{\text{feet}}{\text{minute}}$	the number of feet that the plane climbs each minute	rate of change
$1800t$	feet	the height, in feet, of the plane	output value

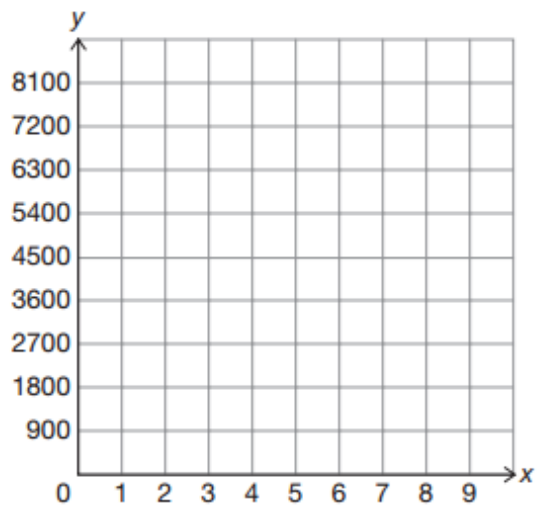
2. Write a function, $h(t)$, to describe the plane's height over time, t.

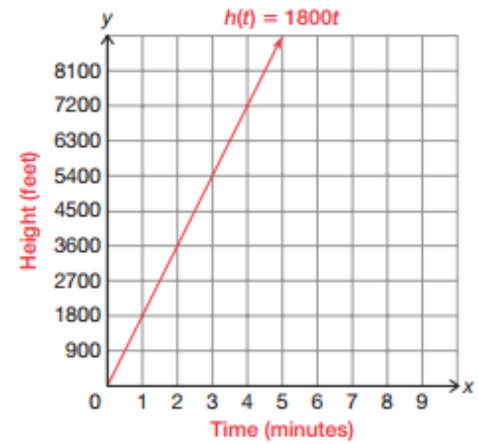
$$h(t) = 1800t$$

3. Which function family does $h(t)$ belong to? Is this what you predicted back in Problem 1, Question 3?

It is a linear function because it is in the form $f(x) = mx + b$. Yes. I predicted that the function would be linear because the plane's ascent is constant over time.

4. Use your table and function to create a graph to represent the change in the plane's height as a function of time. Be sure to label your axes with the correct units of measure and write the function.





a. What is the slope of this graph? Explain how you know.

The equation is $h(t) = 1800t$. The slope of the graph is 1800. In slope-intercept form $y = mx + b$, the slope is given by the coefficient of the x-variable, or m. The slope is the same as the constant unit rate of change, which is $\frac{1800}{1}$.

b. What is the x-intercept of this graph? What is the y-intercept? Explain how you determined each intercept.

The x-intercept and the y-intercept lie at the origin (0,0).
The x-intercept is the point where the graph crosses the x-axis.
The graph crosses the x-axis at $x = 0$.
The y-intercept is the point where the graph crosses the y-axis.
The graph crosses the y-axis at $y = 0$.

c. What do the x- and y-intercepts mean in terms of this problem situation?

The x-intercept represents 0 minutes, the time when the plane takes off.
The y-intercept represents 0 feet, the height of the plane just as it takes off.

Let's consider how to determine the height of the plane, given a time in minutes, using function notation.

To determine the height of the plane at 2 minutes using your function, substitute 2 for *t* every time you see it. Then, simplify the function.

$$h(t) = 1800t$$

Substitute 2 for *t*. ⟶ $h(2) = 1800(2)$

$$h(2) = 3600$$

Two minutes after takeoff, the plane is at 3600 feet.

5. List the different ways the height of the plane is represented in the example.

The different ways the height of the plane is represented are $h(t)$, $h(2)$, 1800(2), and 3600.

6. Use your function to determine the height of the plane at each given time in minutes. Write a complete sentence to interpret your solution in terms of the problem situation.

a. $h(3) =$ 5400 ft

$h(3) = 1800(3)$
$= 5400$

The plane is at 5400 feet 3 minutes after takeoff.

b. $h(3.75) =$ 6750 ft

$h(3.75) = 1800(3.75)$
$= 6750$

The plane is at 6750 feet 3.75 minutes after takeoff.

c. $h(5.1) =$ 9180 ft

$h(5.1) = 1800(5.1)$
$= 9180$

The plane is at 9180 feet 5.1 minutes after takeoff.

d. $h(-4) =$ −7200 ft

$h(-4) = 1800(-4)$
$= -7200$

If I evaluate the function at −4, I get −7200 feet. In terms of the problem situation, −4 would mean 4 minutes ago. In this context, negative height does not make sense. The plane is on the runway at 0 feet.

PROBLEM 3 Connecting Approaches

Now let's consider how to determine the number of minutes the plane has been flying (the input value) given a height in feet (the output value) using function notation.

To determine the number of minutes it takes the plane to reach 7200 feet using your function, substitute 7200 for $h(t)$ and solve.

$$h(t) = 1800t$$

Substitute 7200 for $h(t)$. →

$$7200 = 1800t$$

$$\frac{7200}{1800} = \frac{1800t}{1800}$$

$$4 = t$$

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

1. Why can you substitute 7200 for $h(t)$?

Both 7200 and $h(t)$ represent the height of the plane. Therefore, they are equivalent.

2. Use your function to determine the time it will take the plane to reach each given height in feet. Write a complete sentence to interpret your solution in terms of the problem situation.

a. 5400 feet

$$5400 = 1800t$$
$$\frac{5400}{1800} = \frac{1800t}{1800}$$
$$3 = t$$

The plane will be at 5400 feet 3 minutes after takeoff.

b. 9000 feet

$$9000 = 1800t$$
$$\frac{9000}{1800} = \frac{1800t}{1800}$$
$$5 = t$$

The plane will be at 9000 feet 5 minutes after takeoff.

c. 3150 feet

$$3150 = 1800t$$
$$\frac{3150}{1800} = \frac{1800t}{1800}$$
$$1.75 = t$$

The plane will be at 3150 feet 1.75 minutes after takeoff.

d. 4500 feet

$$4500 = 1800t$$
$$\frac{4500}{1800} = \frac{1800t}{1800}$$
$$2.5 = t$$

The plane will be at 4500 feet 2.5 minutes after takeoff.