2.1

# The Plane!

## Modeling Linear Situations

### LEARNING GOALS

In this lesson, you will:

- Complete tables and graphs, and write equations to model linear situations.
- Analyze multiple representations of linear relationships.
- Identify units of measure associated with linear relationships.
- Determine solutions both graphically and algebraically.
- Determine solutions to linear functions using intersection points.

### KEY TERMS

- first differences
- solution
- intersection point

"Ladies and gentlemen, at this time we ask that all cell phones and pagers be turned off for the duration of the flight. All other electronic devices must be turned off until the aircraft reaches 10,000 feet. We will notify you when it is safe to use such devices."

Flight attendants routinely make announcements like this on airplanes shortly before takeoff and landing. But what's so special about 10,000 feet?

When a commercial airplane is at or below 10,000 feet, it is commonly known as a "critical phase" of flight. This is because research has shown that most accidents happen during this phase of the flight—either takeoff or landing. During critical phases of flight, the pilots and crew members are not allowed to perform any duties that are not absolutely essential to operating the airplane safely.

And it is still not known how much interference cell phones cause to a plane's instruments. So, to play it safe, crews will ask you to turn them off.

## PROBLEM 1 Analyzing Tables

A 747 airliner has an initial climb rate of 1800 feet per minute until it reaches a height of 10,000 feet.

1. Identify the independent and dependent quantities in this problem situation. Explain your reasoning.

   The height of the airplane depends on the time, so height is the dependent quantity and time is the independent quantity.


2. Describe the units of measure for:

   a. the independent quantity (the input values).

      The independent quantity of time is measured in minutes.

   b. the dependent quantity (the output values).

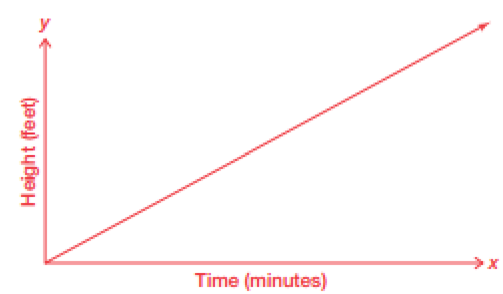
      The dependent quantity of height is measured in feet.

3. Which function family do you think best represents this situation? Explain your reasoning.

   Answers will vary.

   The situation shows a linear function because the rate the plane ascends is constant. So, this situation belongs to the linear function family.

4. Draw and label two axes with the independent and dependent quantities and their units of measure. Then sketch a simple graph of the function represented by the situation.



5. Write the independent and dependent quantities and their units of measure in the table. Then, calculate the dependent quantity values for each of the independent quantity values given.


| | Independent Quantity | Dependent Quantity |
|---|---|---|
| **Quantity** | Time | Height |
| **Units** | minutes | feet |
| | 0 | 0 |
| | 1 | 1800 |
| | 2 | 3600 |
| | 2.5 | 4500 |
| | 3 | 5400 |
| | 3.5 | 6300 |
| | 5 | 9000 |
| **Expression** | $t$ | $1800t$ |


6. Write an expression in the last row of the table to represent the dependent quantity. Explain how you determined the expression.

Each input of time is multiplied by 1800 to produce a height in feet, so the expression for the dependent quantity is $1800t$.

Let's examine the table to determine the unit rate of change for this situation. One way to determine the unit rate of change is to calculate *first differences*. Recall that **first differences** are determined by calculating the difference between successive points.

**7.** Determine the first differences in the section of the table shown.

| | Time (minutes) | Height (feet) | First Differences |
|---|---|---|---|
| | 0 | 0 | |
| 1 − 0 = 1 | | | 1800 − 0 = 1800 |
| | 1 | 1800 | |
| 2 − 1 = 1 | | | 3600 − 1800 = 1800 |
| | 2 | 3600 | |
| 3 − 2 = 1 | | | 5400 − 3600 = 1800 |
| | 3 | 5400 | |

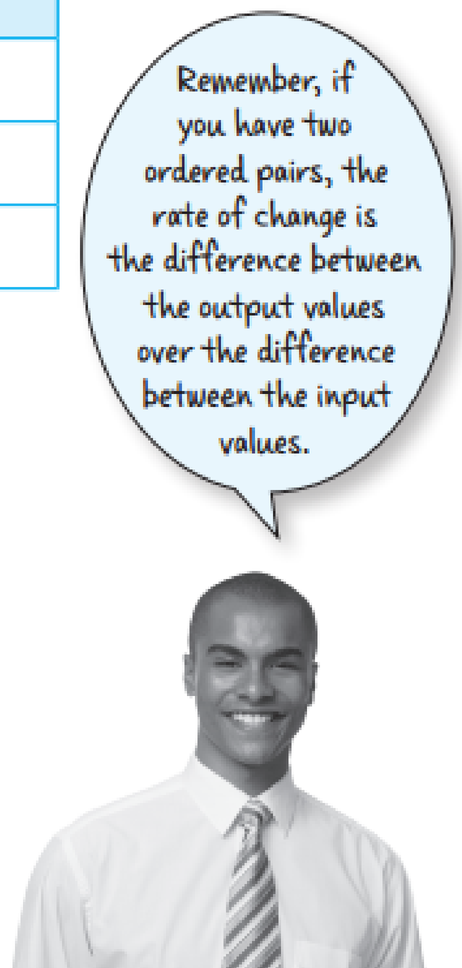
**8.** What do you notice about the first differences in the table? Explain what this means.

The first differences are all the same. They are all 1800. This means that the unit rate of change is 1800 feet per minute.

Another way to determine the unit rate of change is to calculate the rate of change between any two ordered pairs and then write each rate with a denominator of 1.

9. Calculate the rate of change between the points represented by the given ordered pairs in the section of the table shown. Show your work.


| Time (minutes) | Height (feet) |
|---|---|
| 2.5 | 4500 |
| 3 | 5400 |
| 5 | 9000 |




a. (2.5, 4500) and (3, 5400)

$$\frac{5400 - 4500}{3 - 2.5} = \frac{900}{0.5} = \frac{1800}{1}$$

b. (3, 5400) and (5, 9000)

$$\frac{9000 - 5400}{5 - 3} = \frac{3600}{2} = \frac{1800}{1}$$

c. (2.5, 4500) and (5, 9000)

$$\frac{9000 - 4500}{5 - 2.5} = \frac{4500}{2.5} = \frac{1800}{1}$$

**10.** What do you notice about the rates of change?

The rate of change is 1800 feet per minute given any two points in the table.

**11.** Use your answers from Question 7 through Question 10 to describe the difference between a rate of change and a unit rate of change.

The rate of change for the plane's climb can be written as $\frac{1800}{1}$ or $\frac{900}{0.5}$, and so on. The unit rate of change must have a denominator of 1. So, the unit rate of change is $\frac{1800}{1}$.

**12.** How do the first differences and the rates of change between ordered pairs demonstrate that the situation represents a linear function? Explain your reasoning.

Both show that the rate of change in the function is constant. When the rate of change between all points is constant, the ordered pairs will form a straight line when plotted.

**13.** Alita says that in order for a car to keep up with the plane on the ground, it would have to travel at only 20.5 miles per hour. Is Alita correct? Why or why not?

Alita is not correct. It is true that 1800 feet per minute is about 20.5 miles per hour, but this rate compares height to time, not horizontal distance to time. The plane is ascending at about 20.5 miles per hour, but its horizontal speed, or ground speed, is probably much faster.