

PROBLEM 2 Analyzing Equations and Graphs



1. Complete the table shown for the problem situation described in Problem 1, *Analyzing Tables*. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

output value	input value	rate of change
--------------	-------------	----------------

		What It Means	
Expression	Unit	Contextual Meaning	Mathematical Meaning
t	minutes	the time, in minutes, that the plane has been in the air	input value
1800	$\frac{\text{feet}}{\text{minute}}$	the number of feet that the plane climbs each minute	rate of change
$1800t$	feet	the height, in feet, of the plane	output value

2. Write a function, $h(t)$, to describe the plane's height over time, t .

$$h(t) = 1800t$$

3. Which function family does $h(t)$ belong to? Is this what you predicted back in Problem 1, Question 3?

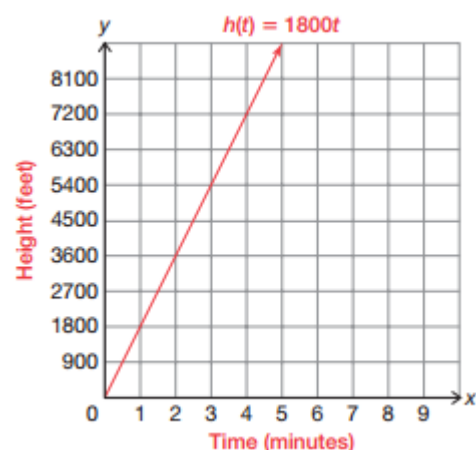
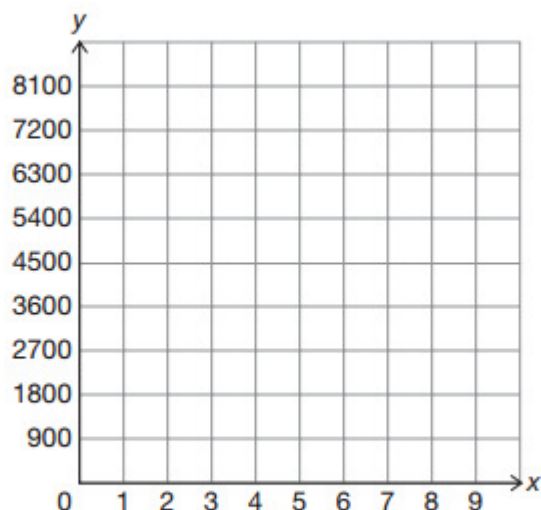
It is a linear function because it is in the form $f(x) = mx + b$.

Yes. I predicted that the function would be linear because the plane's ascent is constant over time.

Why do you think $h(t)$ is used to name this function?



4. Use your table and function to create a graph to represent the change in the plane's height as a function of time. Be sure to label your axes with the correct units of measure and write the function.



- a. What is the slope of this graph? Explain how you know.

The equation is $h(t) = 1800t$. The slope of the graph is 1800. In slope-intercept form $y = mx + b$, the slope is given by the coefficient of the x -variable, or m . The slope is the same as the constant unit rate of change, which is $\frac{1800}{1}$.

- b. What is the x -intercept of this graph? What is the y -intercept? Explain how you determined each intercept.

The x -intercept and the y -intercept lie at the origin $(0,0)$.

The x -intercept is the point where the graph crosses the x -axis.

The graph crosses the x -axis at $x = 0$.

The y -intercept is the point where the graph crosses the y -axis.

The graph crosses the y -axis at $y = 0$.

- c. What do the x - and y -intercepts mean in terms of this problem situation?

The x -intercept represents 0 minutes, the time when the plane takes off.

The y -intercept represents 0 feet, the height of the plane just as it takes off.



Let's consider how to determine the height of the plane, given a time in minutes, using function notation.

To determine the height of the plane at 2 minutes using your function, substitute 2 for t every time you see it. Then, simplify the function.

$$h(t) = 1800t$$

Substitute 2 for t . \longrightarrow $h(2) = 1800(2)$

$$h(2) = 3600$$

Two minutes after takeoff, the plane is at 3600 feet.

5. List the different ways the height of the plane is represented in the example.

The different ways the height of the plane is represented are $h(t)$, $h(2)$, $1800(2)$, and 3600.

6. Use your function to determine the height of the plane at each given time in minutes.
Write a complete sentence to interpret your solution in terms of the problem situation.

a. $h(3) = \frac{5400 \text{ ft}}{\quad}$

$$\begin{aligned} h(3) &= 1800(3) \\ &= 5400 \end{aligned}$$

The plane is at 5400 feet
3 minutes after takeoff.

b. $h(3.75) = \frac{6750 \text{ ft}}{\quad}$

$$\begin{aligned} h(3.75) &= 1800(3.75) \\ &= 6750 \end{aligned}$$

The plane is at 6750 feet
3.75 minutes after takeoff.

c. $h(5.1) = \frac{9180 \text{ ft}}{\quad}$

$$\begin{aligned} h(5.1) &= 1800(5.1) \\ &= 9180 \end{aligned}$$

The plane is at 9180 feet
5.1 minutes after takeoff.

d. $h(-4) = \frac{-7200 \text{ ft}}{\quad}$

$$\begin{aligned} h(-4) &= 1800(-4) \\ &= -7200 \end{aligned}$$

If I evaluate the function at -4 , I get -7200 feet. In terms of the problem situation, -4 would mean 4 minutes ago. In this context, negative height does not make sense. The plane is on the runway at 0 feet.

PROBLEM 3 Connecting Approaches



Now let's consider how to determine the number of minutes the plane has been flying (the input value) given a height in feet (the output value) using function notation.



To determine the number of minutes it takes the plane to reach 7200 feet using your function, substitute 7200 for $h(t)$ and solve.



$$h(t) = 1800t$$



Substitute 7200 for $h(t)$. \longrightarrow $7200 = 1800t$



$$\frac{7200}{1800} = \frac{1800t}{1800}$$



$$4 = t$$



After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

1. Why can you substitute 7200 for $h(t)$?

Both 7200 and $h(t)$ represent the height of the plane. Therefore, they are equivalent.

2. Use your function to determine the time it will take the plane to reach each given height in feet. Write a complete sentence to interpret your solution in terms of the problem situation.

a. 5400 feet

$$5400 = 1800t$$

$$\frac{5400}{1800} = \frac{1800t}{1800}$$

$$3 = t$$

The plane will be at 5400 feet
3 minutes after takeoff.

b. 9000 feet

$$9000 = 1800t$$

$$\frac{9000}{1800} = \frac{1800t}{1800}$$

$$5 = t$$

The plane will be at 9000 feet
5 minutes after takeoff.

c. 3150 feet

$$3150 = 1800t$$

$$\frac{3150}{1800} = \frac{1800t}{1800}$$

$$1.75 = t$$

The plane will be at 3150 feet
1.75 minutes after takeoff.

d. 4500 feet

$$4500 = 1800t$$

$$\frac{4500}{1800} = \frac{1800t}{1800}$$

$$2.5 = t$$

The plane will be at 4500 feet
2.5 minutes after takeoff.

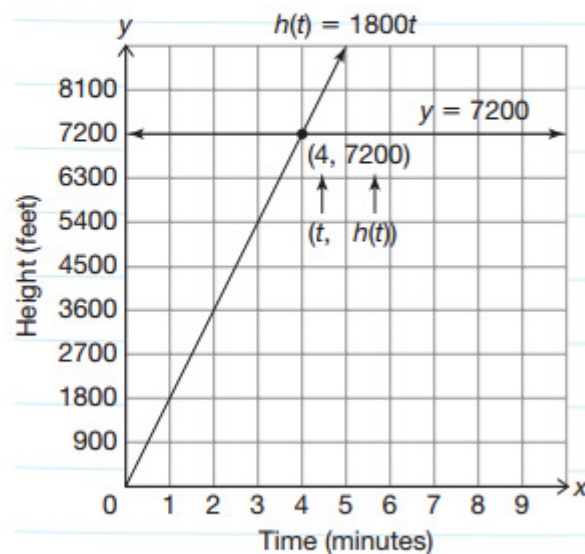
You can also use the graph to determine the number of minutes the plane has been flying (input value) given a height in feet (output value). Remember, the **solution** of a linear equation is any value that makes the open sentence true. If you are given a graph of a function, a solution is any point on that graph. The graph of any function, f , is the graph of the equation $y = f(x)$. If you have intersecting graphs, a solution is the ordered pair that satisfies both functions at the same time, or the **intersection point** of the graphs.

3. What does $(t, h(t))$ represent?

To determine how many minutes it takes for the plane to reach 7200 feet using your graph, you need to determine the intersection of the two graphs represented by the equation $7200 = 1800t$.

The ordered pair $(t, h(t))$ satisfies both equations at the same time. It is the intersection point of the two equations.

First, graph each side of the equation and then determine the intersection point of the two graphs.



$$\begin{array}{rcl}
 h(t) = 1800t & & h(t) = 1800t \\
 7200 = 1800t & & 7200 = 1800t \\
 \downarrow & & \downarrow \\
 y = 7200 & & y = 1800x \\
 \text{Solution: } (4, 7200) & &
 \end{array}$$

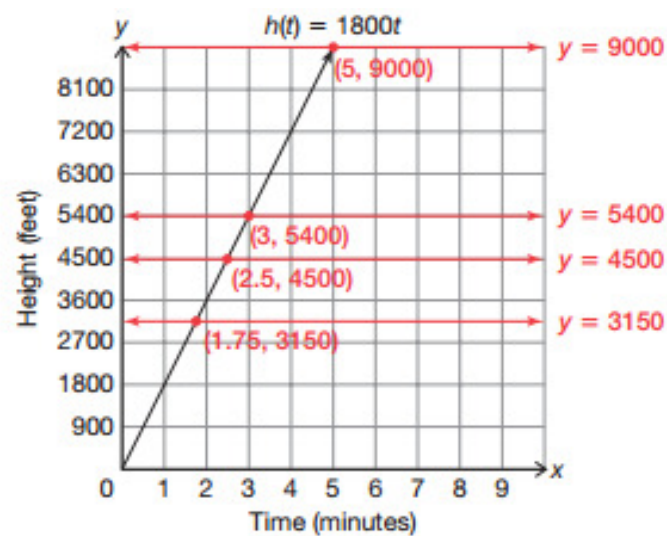
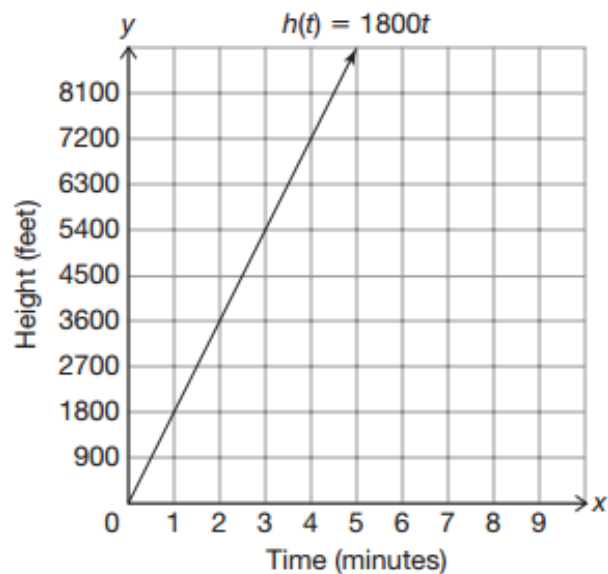
4. Explain the connection between the form of the function $h(t) = 1800t$ and the equation $y = 1800x$ in terms of the independent and dependent quantities.

The dependent quantities are represented by $h(t)$ and y in each form. The independent quantities are represented by t and x in each form.

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

5. Use the graph to determine how many minutes it will take the plane to reach each height.

- a. $h(t) = 5400$
- b. $h(t) = 9000$
- c. $h(t) = 3150$
- d. $h(t) = 4500$



Label all your horizontal lines and the intersection points.



Were you able to get exact answers using the graph?



6. Compare and contrast your solutions using the graphing method to the solutions in Question 2, parts (a) through (d) where you used an algebraic method. What do you notice?

The solutions are the same. Some solutions are hard to read on the graph, but solving the equation for the input value gives me an exact answer.