2.2

What Goes Up Must Come Down

Analyzing Linear Functions

LEARNING GOALS

In this lesson, you will:

- Complete tables and graphs, and write equations to model linear situations.
- Analyze multiple representations of linear relationships.
- Identify units of measure associated with linear relationships.
- Determine solutions to linear functions using intersection points and properties of equality.
- Determine solutions using tables, graphs, and functions.
- Compare and contrast different problem-solving methods.
- Estimate solutions to linear functions.
- Use a graphing calculator to analyze functions and their graphs.

There are 3 ways to model linear equations.

PROBLEM 1 As We Make Our Final Descent - It's time to land our plane!

At 36,000 feet, the crew aboard the 747 airplane begins making preparations to land. The plane descends at a rate of 1500 feet per minute until it lands.

1. Compare this problem situation to the problem situation in Lesson 2.1, *The Plane!* How are the situations the same? How are they different?

See page 78. *Time is still the independent quantity. Height is still the dependent quantity. The rate of change is different because the plane is descending. It will be negative.*

2. Complete the table to represent this problem situation.

	Independent Quantity	Dependent Quantity
Quantity	***Time***	***Height***
Units	***minutes***	***feet***
	0	*36,000*
	2	*33,000*
	4	*30,000*
	6	*27,000*
	12	18,000
	20	6000
Expression	t	***36000 – 1500t***

You are counting by 2's.

How does that affect the dependent quantity?

3. Write a function, g(t), to represent this problem situation.

$g(t) = 36000 - 1500t$

or

$g(t) = -1500t + 36000$

Slope-intercept Form

The rate of change is -1500 feet per minute.

We are losing altitude!

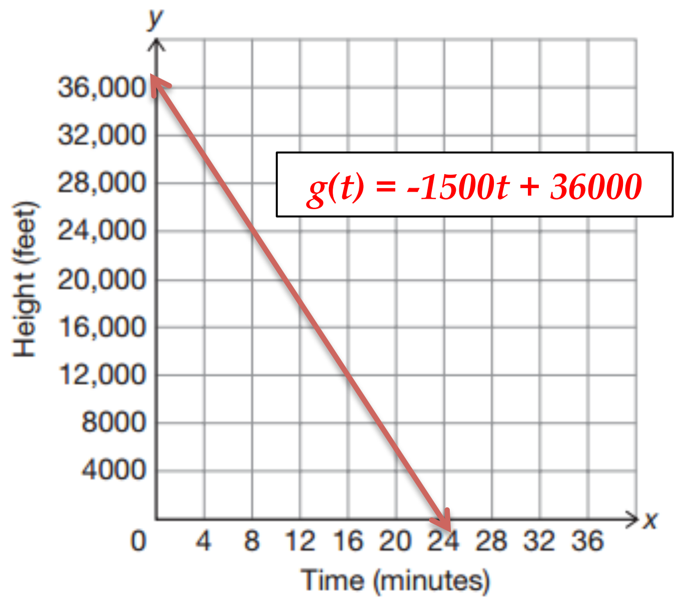
4. Complete the table shown. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

input value	output value	rate of change
y-intercept	x-intercept	

		Description	
Expression	**Units**	**Contextual Meaning**	**Mathematical Meaning**
t	*minutes*	*The amount of time the plane descended*	*input or independent value*
-1500	$\frac{feet}{minute}$	*# of feet the plane descends each minute*	*rate of change or slope*
$-1500t$	*feet*	*# of feet the plane has descended*	
36,000	*feet*	*The plane's initial height*	*y-intercept or starting pt*
$-1500t + 36,000$	*feet*	*The plane's height*	*output or dependent value*

5. Graph $g(t)$ on the coordinate plane shown.

output or
dependent value

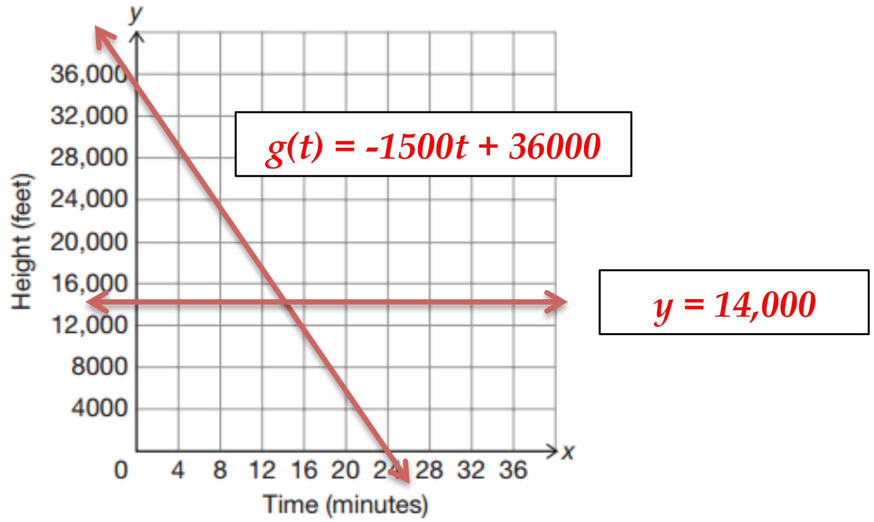


input or independent value

You have just represented the *As We Make Our Final Descent* scenario in different ways:

- numerically, by completing a table,
- algebraically, by writing a function, and
- graphically, by plotting points.

Let's consider how to use each of these representations to answer questions about the problem situation.



6. Determine how long will it take the plane to descend to 14,000 feet.

a. Use the table to determine how long it will take the plane to descend to 14,000 feet.

Look at the table on page 88. *The plane descends to 14,000 feet between 12 and 20 minutes.*

b. Graph and label y = 14,000 on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this problem situation.

Use the graph on page 89. Draw a horizontal line where y = 14,000. *The point-of-intersection is halfway between 12 and 16 minutes, or approximately 14 minutes.*

c. Substitute 14,000 for g(t) and solve the equation for t. Interpret your solution in terms of this problem situation.

$$g(t) = -1500t + 36{,}000$$
$$14{,}000 = -1500t + 36{,}000$$
$$14{,}000 - 36{,}000 = -1500t + 36{,}000 - 36{,}000$$
$$\frac{-22{,}000}{-1500} = \frac{-1500t}{-1500}$$
$$14.667 = t$$

d. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.

Using the table, I can estimate the time. I can use the point-of-intersection on a graph to get an approximate time. I can calculate an exact time using the function or equation.

On your own!

7. Determine how long it will take the plane to descend to 24,000 feet.

a. Use the table to determine how long it will take the plane to descend to 24,000 feet.
Look at the table on page 88. *The plane descends to 24,000 feet between 6 and 12 minutes.*

b. Graph and label y = 24,000 on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this situation.
Use the graph on page 89. Draw a horizontal line where y = 24,000. *The point-of-intersection means the plane is at 24,000 feet at 8 minutes.*

c. Substitute 24,000 for g(t) and solve the equation for t. Interpret your solution in terms of this situation.

$$g(t) = -1500t + 36{,}000$$

$$24{,}000 = -1500t + 36{,}000$$

$$24{,}000 - 36{,}000 = -1500t + 36{,}000 - 36{,}000$$

$$\frac{-12{,}000}{-1500} = \frac{-1500t}{-1500}$$

$$8 = t$$

At 8 minutes, the plane is at 24,000 feet.

d. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.
Using the table, I can only estimate the time. I can use the point-of-intersection on the graph to determine the time because the lines crossed exactly at a marked point on the coordinate plane. I can calculate an exact time using the function.

8. For how many heights can you calculate the exact time using the:

a. table?

Since there are 6 rows of values in the table on page 88, I can use the table to calculate the exact time for 6 different heights.

b. graph?

There are 4 instances where the graph of the function intersects another line or intersects the axes of the coordinate plane so I can use the graph to calculate 4 different exact times.

c. function?

I can solve the function for any given height so I can calculate an infinite number of exact times given the height.

9. Use the word bank to complete each sentence.

always	sometimes	never

If I am given a dependent value and need to calculate an independent value of a linear function,

a. I can ___always___ use a table to determine an *approximate* value.

b. I can ___sometimes___ use a table to calculate an *exact* value.

c. I can ___always___ use a graph to determine an *approximate* value.

d. I can ___sometimes___ use a graph to calculate an *exact* value.

e. I can ___always___ use a function to determine an *approximate* value.

f. I can ___always___ use a function to calculate an *exact* value.