Cool As A Cucumber or Hot Like A Tamale!

3.3

Literal Equations in Standard and Slope-Intercept Form

LEARNING GOALS

In this lesson, you will:

- Recognize and use literal equations.
- Convert literal equations to highlight a specific variable.
- Convert between standard and slope-intercept form.
- Recognize the value of standard and slope-intercept form.

KEY TERMS

- literal equation

Literal equations are equations in which the variables represent specific measures. You most often see literal equations when you study formulas. These literal equations can be manipulated in order to allow you to solve for one specific variable.

A common literal equation is the formula for converting degrees Fahrenheit to degrees Celsius.

$$C = \frac{5}{9}(F - 32)$$

1. The normal temperature for the human body is 98.6°F. What temperature is that in degrees Celsius?

$$C = \frac{5}{9}(98.6 - 32)$$

$$C = \frac{5}{9}(66.6)$$

$$C = 37$$

The normal human body temperature is equivalent to 37°C.

2. The coldest temperature ever recorded on Earth was in Antarctica in 1983. The temperature recorded was −126.8°F. What is this temperature in degrees Celsius?

$$C = \frac{5}{9}\left(-126.8 - 32\right)$$

$$C = \frac{5}{9}\left(-158.8\right)$$

$$C = -88.22$$

$$-88.22°C$$

3. The directions on a box of cake batter tells you to bake your cake at 177°C. What is this temperature in degrees Fahrenheit?

$$177 = \frac{5}{9}(F - 32)$$

$$\frac{9}{5} \cdot 177 = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$$

$$318.6 = F - 32$$

$$350.6 = F \qquad 350.6°F$$

4. Is there a more efficient way to determine degrees Fahrenheit than the method you used in Question 3?

Yes. I could get "F" by itself.

5. Convert the given formula to determine degrees Fahrenheit. Show and explain your work.

$C = \frac{5}{9}(F - 32)$ ⟵ Just rearrange this formula so "F" is by itself!

$$C = \frac{5}{9}(F - 32)$$

$$\frac{9}{5} \cdot C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F \qquad or \qquad \boxed{F = \frac{9}{5}C + 32}$$

6. The hottest temperature ever recorded on Earth occurred in Africa in 1922. It was recorded as 57.8°C. Use your formula to determine this temperature in degrees Fahrenheit.

$F = \frac{9}{5}(57.8) + 32$

$F = 104.04 + 32$

$F \approx 136.04$

7. Dry ice melts at −78°C. At what temperature in degrees Fahrenheit does dry ice melt?

$F = \frac{9}{5}(-78) + 32$

$F = -140.4 + 32$

$F = -108.4$

Convert each given equation to the form indicated.
Then, identify the *x*-intercept, *y*-intercept, and the slope.
Show your work for each.

Remember, to convert to slope-intercept form, solve for *y*. To convert to standard form, get both variables on the same side and the constant on the other.

1. $6x + 5y = 20$

a. slope-intercept form:

$y = mx + b$

$6x + 5y = 20$

$5y = -6x + 20$

$\dfrac{5y}{5} = \dfrac{-6x}{5} + \dfrac{20}{5}$

$y = -\dfrac{6}{5}x + 4$

b. *x*-intercept:

$6x + 5y = 20$

$6x + 5(0) = 20$

$6x = 20$

$x = \dfrac{20}{6}$

c. *y*-intercept:

$y = 4$

d. slope:

$m = -\dfrac{6}{5}$

2. $y = -\frac{2}{3}x + 10$

    a. standard form:

       $Ax + By = C$

       $y = -\frac{2}{3}x + 10$

       $(3)y = 3\left(-\frac{2}{3}x + 10\right)$

       $3y = -2x + 30$

    $2x + 3y = 30$

    b. x-intercept:

       $2x + 3y = 30$

       $2x + 3(0) = 30$

       $2x = 30$

       $x = 15$

    c. y-intercept:

       $y = 10$

    d. slope:

       $m = -\frac{2}{3}$

3. $Ax + By = C$

    a. slope-intercept form:

       $Ax + By = C$

       $By = -Ax + C$

       $\frac{By}{B} = \frac{-Ax}{B} + \frac{C}{B}$

       $y = -\frac{A}{B}x + \frac{C}{B}$

    b. x-intercept:

       $Ax + By = C$

       $Ax + B(0) = C$

       $\frac{Ax}{A} = \frac{C}{A}$

       $x = \frac{C}{A}$

    c. y-intercept:

       $y = \frac{C}{B}$

    d. slope:

       $m = \frac{A}{B}$

4. If you want to determine the y-intercept of an equation, which form is more efficient? Explain your reasoning.

slope-intercept form because it "shows" you the y-intercept.

5. If you want to determine the x-intercept of an equation, which form is more efficient? Explain your reasoning. Answers may vary.

Either works by plugging in "0" for y.

6. If you wanted to graph an equation on your calculator, which form is more efficient? Explain your reasoning.

Either in Desmos is ok.

PROBLEM 3   Other Ways to Write Familiar Formulas

Convert each literal equation to solve for the given variable.

1. Think Inside the Box is manufacturing new boxes for You Pack 'Em, We Ship 'Em (YPEWSE). YPEWSE told Think Inside the Box that the boxes must have a specific volume and area. However, YPEWSE did not specify a height for the boxes.

   a. Write a literal equation to calculate the volume of a box.

   $$V = lwh$$

   b. Convert the volume formula to solve for height.

   $$V = lwh$$

   $$\frac{V}{lw} = \frac{lwh}{lw}$$

   $$\frac{V}{lw} = h$$

   c. YPEWSE specified the volume of the box must be 450 in³ and the area of the base must be 75 in². Use your formula to determine the height of the new boxes.

   $$h = \frac{V}{lw}$$

   $$h = \frac{450}{75}$$

   $$h = 6$$

2. The volume of an ice cream cone is the measure of how much ice cream a cone can hold. An ice cream cone company wants to make an ice cream cone with a larger radius that still holds the same amount of ice cream.

a. Write an equation to calculate the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$

b. Convert the equation to solve for the radius.

$$V = \frac{1}{3}\pi r^2 h$$

$$(3)V = 3\left(\frac{1}{3}\pi r^2 h\right)$$

$$\frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\frac{3V}{\pi h} = r^2$$

$$\sqrt{\frac{3V}{\pi h}} = r$$

3. Future value is the value of a sum of money at a specific date due to interest. The formula $A = P(1 + rt)$ is used to determine future value. The variable A is the future value, P is the principal, r is the interest rate, and t is the time.

A bank wants to know the interest rate of a customer's account who earned a certain amount of future value.

a. Convert the equation to solve for rate.

$$A = P(1 + rt)$$

$$A = P + Prt$$

$$\underline{-P \quad -P}$$

$$A - P = Prt$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$\frac{A - P}{Pt} = r$$

b. Jillian deposited $5000 in an account 10 years ago after her college graduation. The money she deposited now has a value of $15,000. Determine the interest rate of Jillian's account.

$$\frac{A - P}{Pt} = r$$

$$\frac{15,000 - 5000}{5000 \cdot 10} = r$$

$$\frac{10,000}{50,000} = r$$

$$0.2 = r$$

The account had an interest rate of 0.2 or 20%.