4.2

The Password Is… Operations!

Arithmetic and Geometric Sequences

LEARNING GOALS

In this lesson, you will:

- Determine the next term in a sequence.
- Recognize arithmetic sequences.
- Determine the common difference.
- Recognize geometric sequences.
- Determine the common ratio.

KEY TERMS

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

Work with a partner or by yourself on pages 225 and 227.

1. Find each pattern.
2. Continue the sequence in the blanks provided.

You have 10 minutes!

A

45, 90, 180, 360, 720, 1440, 2880, . . .

multiply by 2

B

−4, −2, 0, 2, 4, 6, 8, . . .

add 2

C

−2, −6, −18, −54, −162, −486, −1458, . . .

multiply by 3

D

2, 5, 10, 17, 26, 37, 50, . . .

add 3, then 5, then 7, etc.

E

$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}, \underline{-5}, \underline{-\frac{29}{4}}, \underline{-\frac{19}{2}}, \ldots$

subtract $\frac{9}{4}$

Hint: find a common denominator

F

1234, 123.4, 12.34, 1.234, 0.1234, 0.01234, 0.001234, . . .

multiply by 0.1 or divide by 10

G

1, −2, 3, −4, 5, −6, 7, −8, 9, . . .

consecutive #s, but every other one is negative.

H

−20, −16, −12, −8, −4, 0, 4, 8, . . .

add 4

I

1, 10, 100, 1000, 10,000 , 100,000 , . . .

multiply by 10

J

$-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8}, -\frac{5}{16}, -\frac{5}{32}, \ldots$

multiply by $\frac{1}{2}$

K

6.5, 5, 3.5, 2, 0.5 , −1 , −2.5 , . . .

subtract 1.5

L

86, 85, 83, 80, 76, 71 , 65 , . . .

subtract 1, then 2, then 3, . . .

M

$-16, 4, -1, \frac{1}{4}, \underline{-\frac{1}{16}}, \underline{\frac{1}{64}}, \ldots$

divide by -4

N

1473.2, 1452.7, 1432.2, 1411.7, <u>1391.2</u>,

<u>1370.7</u>, <u>1350.2</u>, . . .

subtract 20.5

O

$\sqrt{5}, 2, \sqrt{3}, \sqrt{2}, 1, 0, \sqrt{-1}$ <u>$\sqrt{-2}$</u>,

<u>$\sqrt{-3}$</u>, . . .

square roots of consecutive integers

P

$-4, 12, -36, 108,$ <u>-324</u>, <u>972</u>, . . .

multiply by -3

PROBLEM 2 Arithmetic, My Dear Watson!

You can describe a pattern as adding a constant to, or subtracting a constant from each term to determine the next term for some sequences. For other sequences, you can describe the pattern as multiplying or dividing each term by a constant to determine the next term. Still other sequences cannot be described either way.

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a positive or negative constant is added to each term to produce the next term. This positive or negative constant is called the **common difference**. The common difference is typically represented by the variable d.

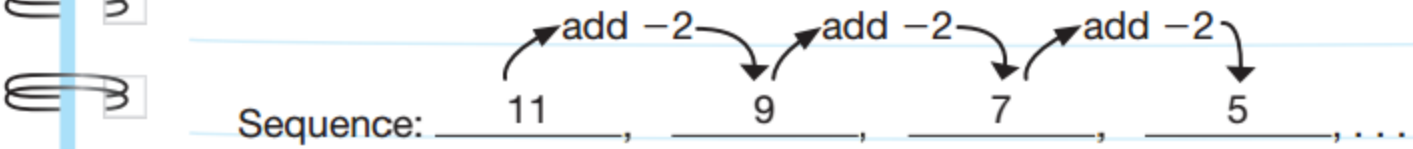
6, 8, 10, 12,…
You add 2 each time so $d = 2$.

5, 3, 1, -1,…
You subtract 2 each time so $d = -2$.

Consider the sequence shown.

11, 9, 7, 5, ...

The pattern is to add the same negative number, −2, to each term to determine the next term.



This sequence is arithmetic and the common difference d is −2.

If the difference between each number in the sequence is the SAME or CONSTANT, then it's arithmetic.

To find d, take the 2nd number and subtract the 1st number.
9 – 11 = -2, so d = -2.

1. Suppose a sequence has the same starting number as the sequence in the worked example, but its common difference is 4.

   a. How would the pattern change?

   Start with 11, but change the common difference from -2 to 4.

   The sequence would increase by 4 instead of decreasing by 2.

   b. Is the sequence still arithmetic? Why or why not?

   Yes, because you keep adding 4 each time. It has a "constant" difference.

   c. If possible, write the first 5 terms of the new sequence.

   11, 15, 19, 23, 27

2. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know?* Go back to page 225 and look at each sequence.

   a. List those sequences that are arithmetic.

B, E, H, K, N

Now let's look at each of those and find the common difference.

B

$-4, -2, 0, 2, \underline{4}, \underline{6}, \underline{8}, \ldots$

add 2

$-2 - (-4) = -2 + 4 = 2$

Arithmetic: $d = 2$

E

$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}, \underline{-5}, \underline{-\frac{29}{4}}, \underline{-\frac{19}{2}}, \ldots$

subtract $\frac{9}{4}$

$7/4 - 4 = 7/4 - 16/4 = -9/4$

Arithmetic: $d = -9/4$

H

−20, −16, −12, −8, −4, 0 ,

4 , 8 , . . .

add 4

Arithmetic: $d = 4$

K

6.5, 5, 3.5, 2, 0.5 , −1 ,

−2.5 , . . .

subtract 1.5

Arithmetic: $d = -1.5$

N

1473.2, 1452.7, 1432.2, 1411.7, 1391.2,

1370.7 , 1350.2 , . . .

subtract 20.5

Arithmetic: $d = -20.5$