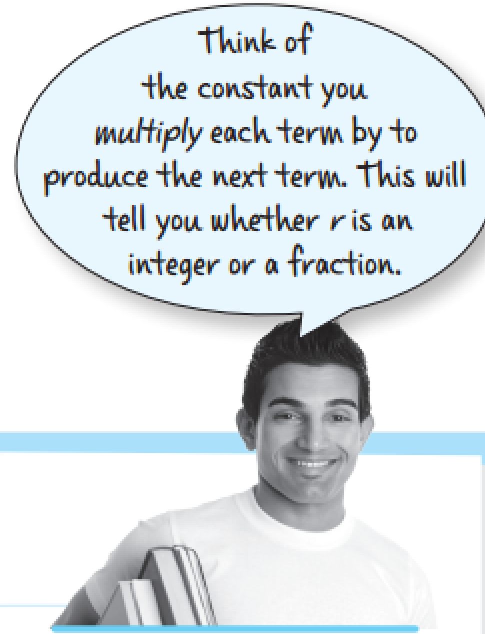
A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the **common ratio**. The common ratio is represented by the variable *r*.



Consider the sequence shown.

1, 2, 4, 8, . . .

The pattern is to multiply each term by the same number, 2, to determine the next term.

$$\text{Sequence: } 1 \xrightarrow{\text{multiply by 2}} 2 \xrightarrow{\text{multiply by 2}} 4 \xrightarrow{\text{multiply by 2}} 8, \ldots$$

This sequence is geometric and the common ratio *r* is 2.

If the ratio between each number in the sequence is the SAME or CONSTANT, then it's geometric.

To find *r*, you can always take the 2nd number and divide by the 1st number.
$2 \div 1 = 2$, so $r = 2$.

3. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is 3. Start with 1, but change the common ratio from 2 to 3.

a. How would the pattern change?

The sequence would still increase, but the terms would be different.
The sequence would increase more rapidly.

b. Is the sequence still geometric? Explain your reasoning.

Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

c. If possible, write the first 5 terms for the new sequence.

1, 3, 9, 27, 81

4. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is $\frac{1}{3}$. Start with 1, but change the common ratio to 1/3.

a. How would the pattern change?

The sequence would decrease.

b. Is the sequence still geometric? Why or why not?

Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

c. If possible, write the first 6 terms for the new sequence.

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}$

5. Suppose a sequence has the same starting number as the sequence in the worked example, but its common ratio is -2. Start with 1, but change the common ratio to -2.

a. How would the pattern change?

The sequence would alternatively increase and decrease because the sign would change for every other number.

b. Is the sequence still geometric? Explain your reasoning.

Yes, because you still have a constant ratio between any two numbers.

c. If possible, write the first 6 terms for the new sequence.

$1, -2, 4, -8, 16, -32$

Skip #6.

7. Analyze the sequences you cut out in Problem 1, *What Comes Next, and How Do You Know?* again. Go back to page 225 and look at each sequence.

a. List those sequences that are geometric.

A, C, F, I, J, M, P.

Now let's look at each of those and find the common ratio.

A

45, 90, 180, 360, 720, 1440, 2880, . . .

multiply by 2

$$\frac{90}{45} = 2$$

geometric: $r = 2$

C

−2, −6, −18, −54, −162, −486, −1458, . . .

multiply by 3

$$\frac{-6}{-2} = 3$$

geometric: $r = 3$

F

1234, 123.4, 12.34, 1.234, 0.1234, 0.01234, 0.001234, . . .

multiply by 0.1

$$\frac{123.4}{1234} = 0.1$$

geometric: $r = 0.1$

I

1, 10, 100, 1000, 10,000, 100,000, . . .

multiply by 10

$$\frac{10}{1} = 10$$

geometric: $r = 10$

J

$-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8}, \underline{-\frac{5}{16}}, \underline{-\frac{5}{32}}, \ldots$

multiply by $\frac{1}{2}$

$$-\frac{5}{2} \div -\frac{5}{1}$$

$$-\frac{5}{2} \times -\frac{1}{5} = \frac{1}{2}$$

geometric: $r = \frac{1}{2}$

M

$-16, 4, -1, \frac{1}{4}, \underline{-\frac{1}{16}}, \underline{\frac{1}{64}}, \ldots$

divide by -4

$$\frac{4}{-16} = -\frac{1}{4}$$

geometric: $r = -\frac{1}{4}$

P

$-4, 12, -36, 108, \underline{-324}, \underline{972}, \ldots$

multiply by -3

$$\frac{12}{-4} = -3$$

geometric: $r = -3$

8. Consider the sequences from Problem 1 that are neither arithmetic nor geometric.

a. List these sequences. Go back to page 225 and look at each sequence.

D, G, L, O

b. Explain why these sequences are neither arithmetic nor geometric.

These sequences are neither arithmetic nor geometric because there is no common difference or common ratio for any of these sequences.