4.3

The Power of Algebra Is a Curious Thing

Using Formulas to Determine Terms of a Sequence

LEARNING GOALS

In this lesson, you will:

- Write an explicit formula for arithmetic and geometric formulas.
- Write a recursive formula for arithmetic and geometric formulas.
- Use formulas to determine unknown terms of a sequence.

KEY TERMS

- index
- explicit formula
- recursive formula

PROBLEM 1 Can I Get a Formula?

While a common ratio or a common difference can help you determine the next term in a sequence, how can they help you determine the thousandth term of a sequence? The ten-thousandth term of a sequence? Consider the sequence represented in the given problem scenario.

1. Rico owns a sporting goods store. He has agreed to donate $125 to the Centipede Valley High School baseball team for their equipment fund. In addition, he will donate $18 for every home run the Centipedes hit during the season. The sequence shown represents the possible dollar amounts that Rico could donate for the season.

125, 143, 161, 179, . . .

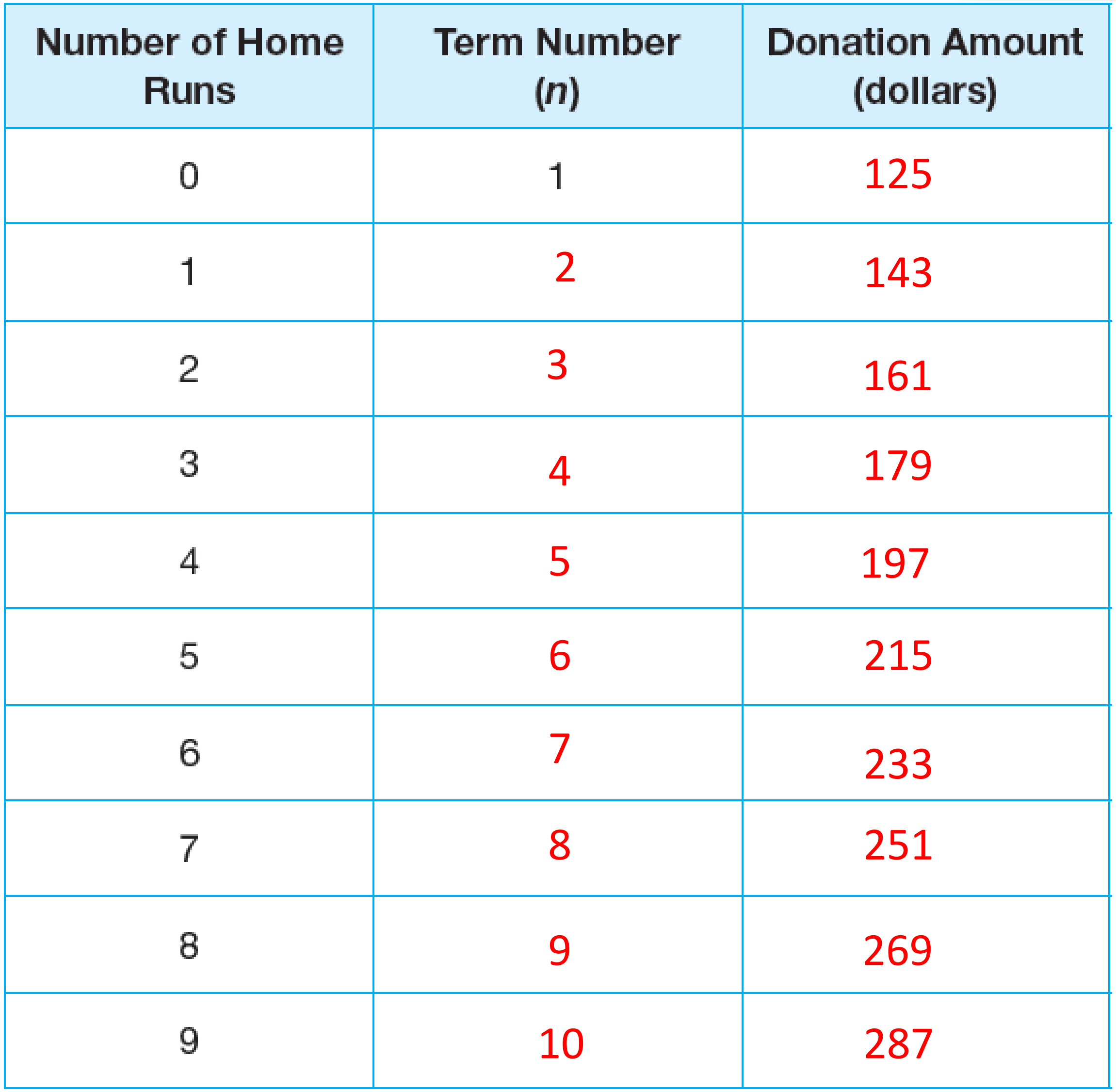
a. Identify the sequence type. Describe how you know.

143 – 125 = 18 and 161 – 143 = 18
It's an arithmetic sequence because you + 18 each time.

b. Determine the common ratio or common difference for the given sequence.

The **common difference**, $d = 18$.

c. Complete the table of values. Use the number of home runs the Centipedes could hit to identify the term number, and the total dollar amount Rico could donate to the baseball team.



Number of Home Runs	Term Number (*n*)	Donation Amount (dollars)
0	1	125
1	2	143
2	3	161
3	4	179
4	5	197
5	6	215
6	7	233
7	8	251
8	9	269
9	10	287

Add 18 each time.

This process is so tedious. There's got to be an easier way!

d. Explain how you can calculate the tenth term based on the ninth term.

+ 18 to the 9th term

e. Determine the 20th term. Explain your calculation.

20th term = 467. Simply, + 18 to the 19th term.

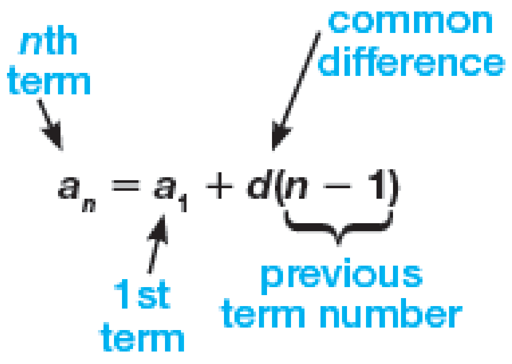
f. Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.

Starting point = \$125, since the # of home runs = 0
ROC (rate of change) = \$18/home run
Let x = the # of home runs. So, use $18x + 125$. (slope-intercept form) ☺

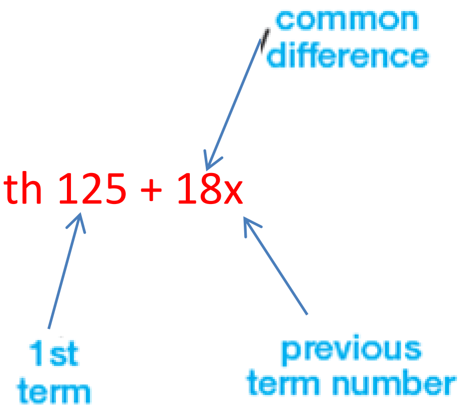
g. Describe a strategy to calculate the 93rd term.

The 93rd term means 92 home runs.
Using $18x + 125 = 18(92) + 125 = 1781$
We've created an explicit formula!

An **explicit formula** of a sequence is a formula for calculating the value of each term of a sequence using the term's position in the sequence. The explicit formula for determining the *n*th term of an arithmetic sequence is:

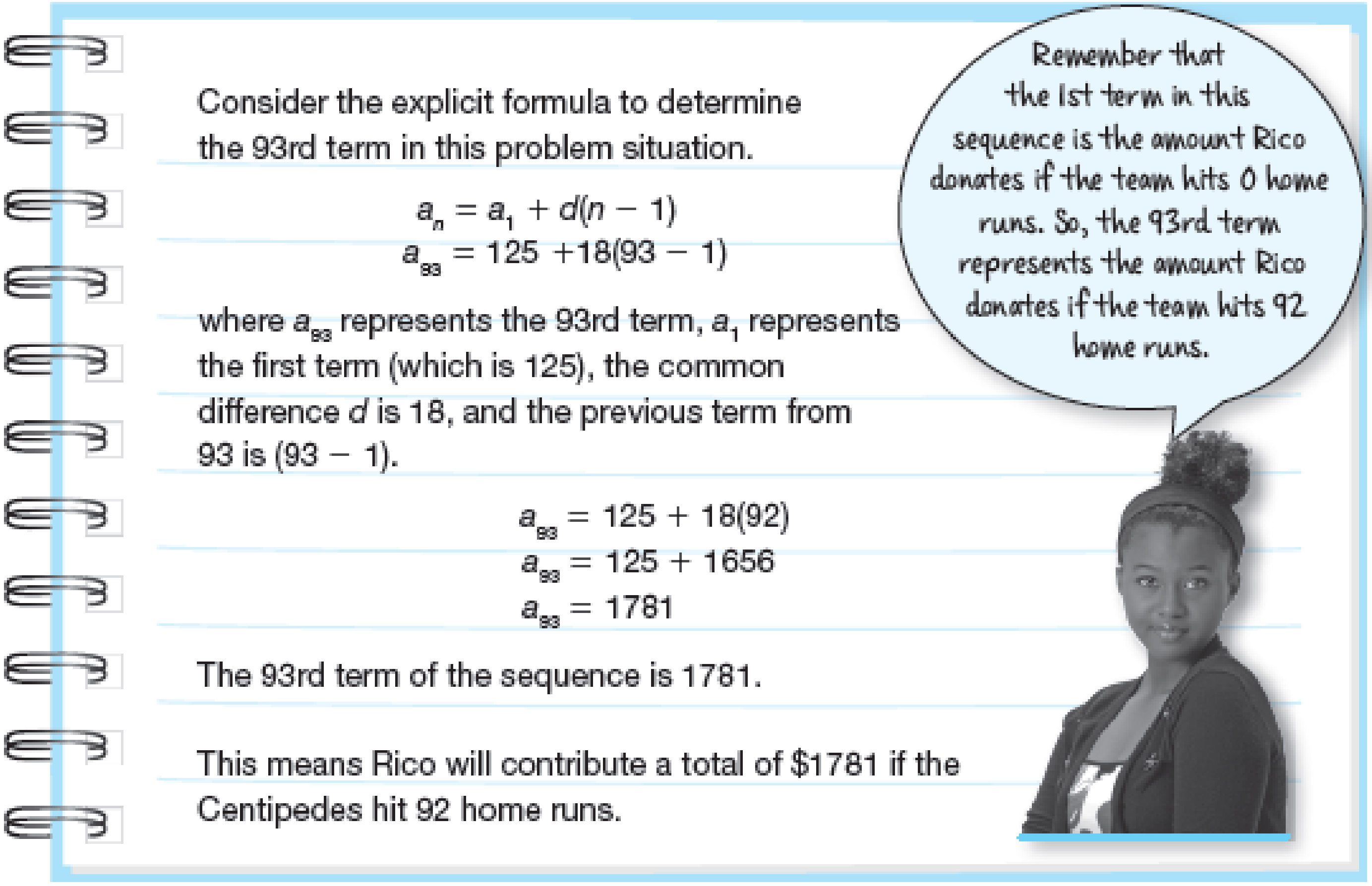


$$a_n = a_1 + d\underbrace{(n-1)}$$



Compare this to what we just did with 125 + 18x

Let’s look at the problem we just solved.



Consider the explicit formula to determine the 93rd term in this problem situation.

$$a_n = a_1 + d(n - 1)$$
$$a_{93} = 125 + 18(93 - 1)$$

where a_{93} represents the 93rd term, a_1 represents the first term (which is 125), the common difference d is 18, and the previous term from 93 is (93 − 1).

$$a_{93} = 125 + 18(92)$$
$$a_{93} = 125 + 1656$$
$$a_{93} = 1781$$

The 93rd term of the sequence is 1781.

This means Rico will contribute a total of $1781 if the Centipedes hit 92 home runs.

Remember that the 1st term in this sequence is the amount Rico donates if the team hits 0 home runs. So, the 93rd term represents the amount Rico donates if the team hits 92 home runs.

$$a_n = a_1 + d(n-1)$$

3. Use the explicit formula to determine the amount of money Rico will contribute if the Centipedes hit:

a. 35 home runs. (36*th term*!)

$$a_{36} = 125 + 18(36-1)$$
$$a_{36} = 125 + 18(35)$$
$$a_{36} = \$755$$

b. 48 home runs.

$$a_{49} = 125 + 18(49-1)$$
$$a_{49} = 125 + 18(48)$$
$$a_{49} = \$989$$

c. 86 home runs.

$$a_{87} = 125 + 18(87-1)$$
$$a_{87} = 125 + 18(86)$$
$$a_{87} = \$1673$$

d. 214 home runs.

$$a_{215} = 125 + 18(215-1)$$
$$a_{215} = 125 + 18(214)$$
$$a_{215} = \$3977$$

$$a_n = a_1 + d(n-1) \longrightarrow a_n = 500 + 75(n-1)$$

4. Rico decides to increase his initial contribution and amount donated per home run hit. He decides to contribute $500 and will donate $75.00 for every home run the Centipedes hit. Determine Rico's contribution if the Centipedes hit:

a. 11 home runs.

$$a_{12} = 500 + 75(12-1)$$

$$a_{12} = 500 + 75(11)$$

$$a_{12} = \$1325$$

b. 26 home runs.

$$a_{27} = 500 + 75(27-1)$$

$$a_{27} = 500 + 75(26)$$

$$a_{27} = \$2450$$

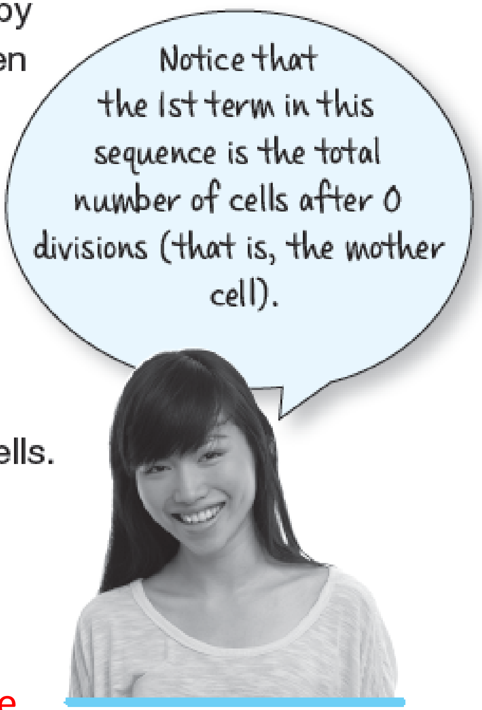
~~**c.** 39 home runs.~~

~~**d.** 50 home runs.~~

Let's Practice!

PROBLEM 2 They're Just Out of Control—But That's A Good Thing!

When it comes to bugs, bats, spiders, and—ugh, any other creepy crawlers—finding one in your house is finding *one* too many! Then again, when it comes to cells, the more the better! Animals, plants, fungi, slime, molds, and other living creatures consist of eukaryotic cells. During growth, generally there is a cell called a "mother cell" that divides itself into two "daughter cells." Each of those daughter cells then divides into two more daughter cells, and so on.



1. The sequence shown represents the growth of eukaryotic cells.

1, 2, 4, 8, 16, . . .

   a. Describe why this sequence is geometric.

2/1 = 2 and 4/2 = 2

It's a geometric sequence because you × 2 each time.

b. Determine the common ratio for the given sequence.

The common ratio, $r = 2$.

c. Complete the table of values. Use the number of cell divisions to identify the term number, and the total number of cells after each division.

Number of Cell Divisions	Term Number (*n*)	Total Number of Cells
0	1	1
1	2	2
2	3	4
3	4	8
4	5	16
5	6	32
6	7	64
7	8	128
8	9	256
9	10	512

Multiply by 2 each time.

This process is so tedious. There's got to be an easier way!

d. Explain how you can calculate the tenth term based on the ninth term.

Multiply the 9th term by 2

e. Determine the 20th term. Explain your calculation.

20th term = 524,288. Simply, continue the pattern and keep multiplying by 2.

f. Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.

Yes. The formula is 2^x, where x represents the # of cell divisions or term # - 1.

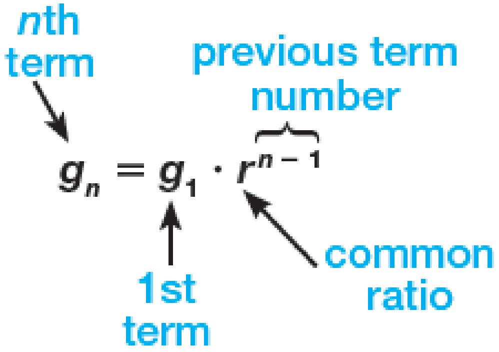
b. Determine the common ratio for the given sequence.

The common ratio, $r = 2$.

c. Complete the table of values. Use the number of cell divisions to identify the term number, and the total number of cells after each division.

Number of Cell Divisions	Term Number (n)	Total Number of Cells
0	1	$1 = 2^0$
1	2	$2 = 2^1$
2	3	$4 = 2^2$
3	4	$8 = 2^3$
4	5	$16 = 2^4$
5	6	$32 = 2^5$
6	7	$64 = 2^6$
7	8	$128 = 2^7$
8	9	$256 = 2^8$
9	10	$512 = 2^9$

The explicit formula for determining the *n*th term of a geometric sequence is:



$$g_n = g_1 \cdot r^{\overbrace{n-1}}$$

common ratio

Compare this to what we just did with $(1)2^x$

1st term

previous term number

$$g_n = g_1 \bullet r^{n-1}$$

3. Use the explicit formula to determine the total number of cells after:

a. 11 divisions.

$$g_{12} = 1 \bullet 2^{12-1}$$
$$g_{12} = 1 \bullet 2^{11}$$
$$g_{12} = 2^{11}$$
$$g_{12} = 2048$$

b. 14 divisions.

$$g_{15} = 1 \bullet 2^{15-1}$$
$$g_{15} = 1 \bullet 2^{14}$$
$$g_{15} = 2^{14}$$
$$g_{15} = 16,384$$

c. ~~18 divisions.~~

d. ~~22 divisions.~~

$$g_n = g_1 \bullet r^{n-1} \longrightarrow \begin{matrix} 1st\ Term = 5 \\ r = 3 \end{matrix} \longrightarrow g_n = 5 \bullet 3^{n-1}$$

4. Suppose that a scientist has 5 eukaryotic cells in a petri dish. She wonders how the growth pattern would change if each mother cell divided into 3 daughter cells. For this situation, determine the total number of cells in the petri dish after:

a. 4 divisions.

$$g_5 = 5 \bullet 3^{5-1}$$
$$g_5 = 5 \bullet 3^{4}$$
$$g_5 = 5 \bullet 81$$
$$g_5 = 405$$

b. 7 divisions.

$$g_8 = 5 \bullet 3^{8-1}$$
$$g_8 = 5 \bullet 3^{7}$$
$$g_8 = 5 \bullet 2187$$
$$g_8 = 10,935$$

~~**c.** 13 divisions.~~

~~**d.** 16 divisions.~~

Let’s Practice!

PROBLEM 3 So, You've Explicitly Determined Terms, But Is There Another Way?

The explicit formula is very handy for determining terms of a sequence, but is there another way?

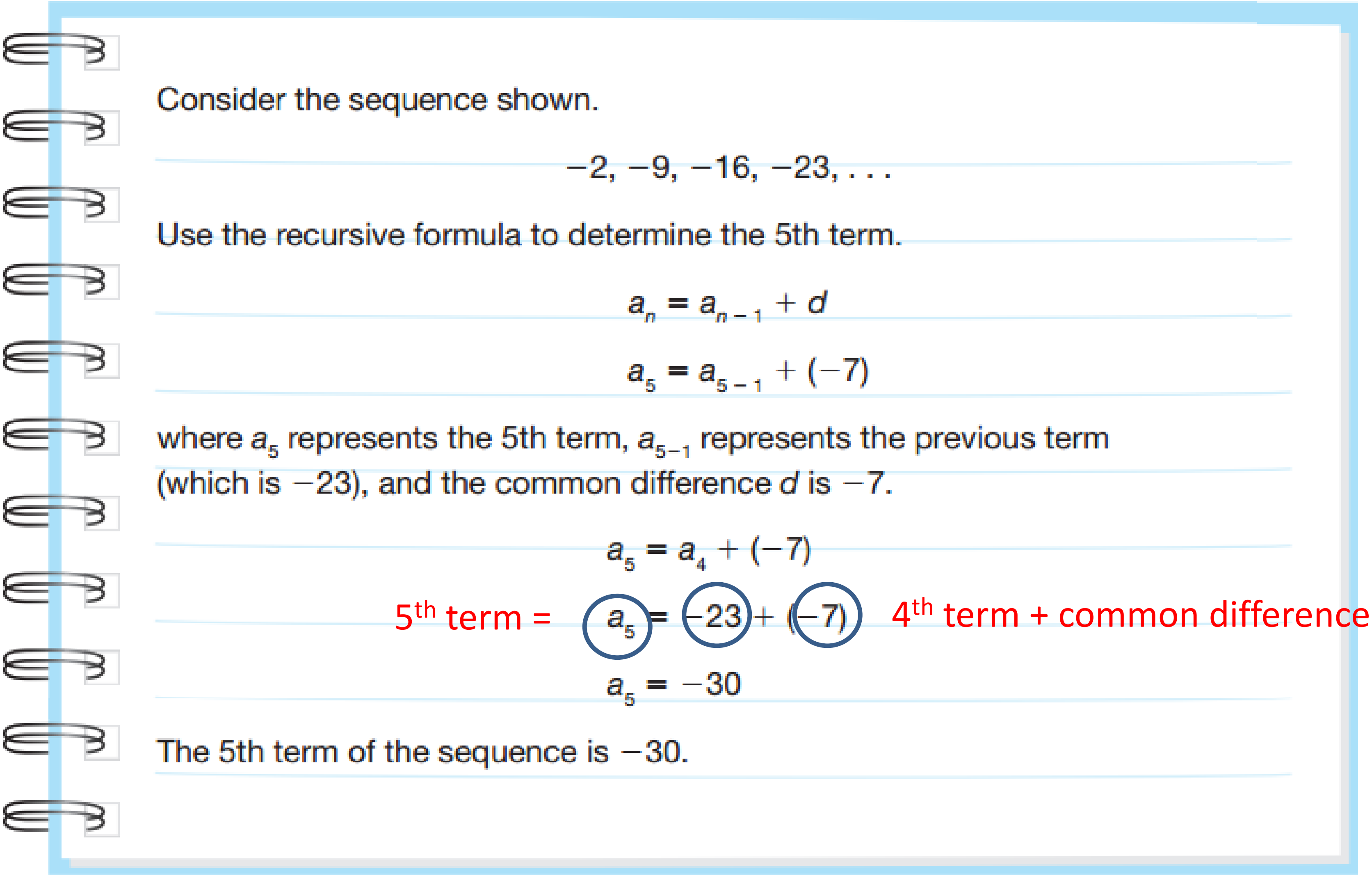
A **recursive formula** expresses each new term of a sequence based on the preceding term in the sequence. The recursive formula for determining the *n*th term of an arithmetic sequence is:

$$\overset{n\text{th term}}{a_n} = \underbrace{a_{n-1}}_{\text{previous term}} + \overset{\text{common difference}}{d}$$

This is basically how we started out… finding the next term in an arithmetic sequence. (EASY!!!) 🙂

Notice that you do not need to know the first term when using the recursive formula. However, you need to know the previous term to determine the next term. This is why this formula is commonly referred to as the NOW NEXT formula.

It looks scary  …but it's just understanding "notation."



Consider the sequence shown.

$$-2, -9, -16, -23, \ldots$$

Use the recursive formula to determine the 5th term.

$$a_n = a_{n-1} + d$$

$$a_5 = a_{5-1} + (-7)$$

where a_5 represents the 5th term, a_{5-1} represents the previous term (which is -23), and the common difference d is -7.

$$a_5 = a_4 + (-7)$$

5th term = $a_5 = -23 + (-7)$ 4th term + common difference

$$a_5 = -30$$

The 5th term of the sequence is -30.

The recursive formula for determining the *n*th term of a geometric sequence is:

*n*th term

common ratio

$$g_n = \underbrace{g_{n-1}}_{\text{previous term}} \cdot r$$

Consider the sequence shown.

4, 12, 36, 108, . . .

Use the recursive formula to determine the 5th term.

$$g_n = g_{n-1} \cdot r$$

$$g_5 = g_{5-1} \cdot (3)$$

where g_5 represents the 5th term, g_{5-1} represents the previous term (which is 108), and the common ratio r is 3.

5th term = $g_5 = g_4 \cdot (3)$ 4th term x common ratio

$$g_5 = 108 \cdot (3)$$

$$g_5 = 324$$

The 5th term of the sequence is 324.

1. Determine whether each sequence is arithmetic or geometric. Then use the recursive formula to determine the unknown term in each sequence.

a. $\frac{5}{3}$, 5, 15, 45, ________ , . . .

geometric; 135

$g_5 = g_4 \cdot r$

$g_5 = 45 \cdot 3$

$g_5 = 135$

b. −45, −61, −77, −93, ________ , . . .

arithmetic; −109

$a_5 = a_4 + d$

$a_5 = -93 + (-16)$

$a_5 = -109$

c. −3, 1, ________ 9, 13, . . .

arithmetic; 5

$a_3 = a_2 + d$

$a_3 = 1 + 4$

$a_3 = 5$

d. −111, 222, ________, 888, −1776, . . .

geometric; −444

$g_3 = g_2 \cdot r$

$g_3 = 222 \cdot -2$

$g_3 = -444$

e. −30, −15, ________, −3.75, −1.875, ________, . . .

geometric; −7.5, −0.9375

$g_3 = g_2 \cdot r$

$g_3 = -15 \cdot \frac{1}{2}$

$g_3 = -7.5$

f. 3278, 2678, 2078, ________, ________, ________, . . .

arithmetic; 1478, 878, 278

$a_4 = a_3 + d$

$a_4 = 2078 + (-600)$

$a_4 = 1478$

2. Consider the sequence in Question 1 part (f).

a. Use the recursive formula to determine the 9th term.

$a_7 = 278 + (-600) = -322$
$a_8 = -322 + (-600) = -922$
$a_9 = -922 + (-600) = -1522$

b. Use the explicit formula to determine the 9th term.

$a_9 = 3278 + (-600)(8) = -1522$

c. Which formula do you prefer? Why?

The explicit formula…It's faster because there are less calculations!

d. Which formula would you use if you wanted to determine the 61st term of the sequence? Explain your reasoning.

The explicit formula… the recursive formula would take forever!