5.2

Downtown and Uptown

Graphs of Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Solve exponential functions using the intersection of graphs.
- Analyze asymptotes of exponential functions and their meanings in context.
- Identify the domain and range of exponential functions.
- Analyze and graph decreasing exponential functions.
- Compare graphs of linear and exponential functions through intercepts, asymptotes, and end behavior.

KEY TERM

- horizontal asymptote

PROBLEM 1 Downtown and Uptown

At this moment, the population of Downtown is 20,000, and the population of Uptown is 6000. But over many years, people have been moving away from Downtown at a rate of 1.5% every year. At the same time, Uptown's population has been growing at a rate of 1.8% each year.

1. What are the independent and dependent quantities in each situation?

   Independent Quantity (IQ): time (years)   Dependent Quantity (DQ): population

2. Which city's population can be represented as an increasing function, and which can be represented as a decreasing function?

   Uptown: Increasing function   Downtown: Decreasing function

Let's analyze the population growth of Uptown. In 1 year from now, the population of Uptown will be

$$6000 + 6000(0.018) = 6108.$$

The population will be 6108 people in Uptown 1 year from now.

3. Write and simplify an expression that represents the population of Uptown:

   a. 2 years from now.

   $6108 + 6108(0.018) = 6217.944$

   The population will be about 6218 people 2 years from now.

   b. 3 years from now.

   $6217.944 + 6217.944(0.018) = 6329.866992$

   The population will be about 6330 in 3 years from now.

4. How can you tell that this function is an exponential function? Explain your reasoning.

Uptown's population is not growing at a constant, but it is increasing each year.

1^{st} year: 6108 – 6000 = 108 more people

2^{nd} year: 6218 – 6108 = 110 more people

3^{rd} year: 6330 – 6218 = 112 more people

You can use the formula for compound interest to determine the function for Uptown's increasing population. Recall that the formula for compound interest is $P(t) = P(1 + r)^t$, where $P(t)$ represents the amount in the account after a certain amount of time in years, r is the interest rate written as a decimal, and t is the time in years.

5. In the compound interest formula, substitute Uptown's starting population for P and the rate of population growth for r.

   a. Write the function, $U(t)$, showing Uptown's population growth as a function of time in years.

   $U(t) = 6000(1+0.018)^t$ $\quad$ $U(t) = 6000(1.018)^t$

   Add when the population increases.

   b. Use your answers to Question 3 and a calculator to verify that your function is correct.

   $U(t) = 6000(1.018)^t$

   $U(t) = 6000(1.018)^2$ $\quad$ $U(t) = 6000(1.018)^3$

   $U(t) \approx 6218$ $\quad$ $U(t) \approx 6330$

Now let's analyze the population decline of Downtown.

6. Write and simplify an expression that represents the population of Downtown. The first one has been done for you.

a. 1 year from now.

$20{,}000 - 20{,}000(0.015) = 19{,}700$

The population of Downtown will be 19,700 people 1 year from now.

b. 2 years from now.

$19{,}700 - 19{,}700(0.015) = 19{,}404.5$

The population of Downtown will be about 19,405 people 2 years from now.

c. 3 years from now.

$19{,}404.5 - 19{,}404.5(0.015) = 19{,}113.4325$

The population of Downtown will be about 19,113 people 3 years from now.

7. Rewrite the expressions for the population decline in Downtown using the Distributive Property. The first one has been done for you.

   a. 1 year from now.

   $20{,}000 - 20{,}000(0.015)$

   $20{,}000(1 - 0.015)$

   b. 2 years from now.

   $19{,}700 - 19{,}700(0.015)$

   $19{,}700(1 - 0.015)$

   c. 3 years from now.

   $19{,}404.5 - 19{,}404.5(0.015)$

   $19{,}404.5(1 - 0.015)$

8. Use the compound interest formula and your expressions in Question 7 to write the function, $D(t)$, showing Downtown's population decline as a function of time in years.

   $D(t) = 20{,}000(1 - 0.015)^t$

Subtract when the population decreases.

9. Think about each function as representing a sequence. What is the common ratio in simplest form, or the number that is multiplied each time to get the next term, in each sequence?

For Uptown, the common ratio is $1 + 0.018$, or 1.018. For Downtown, the common ratio is $1 - 0.015$, or 0.985.

10. Explain how the common ratios determine whether the exponential functions for the change in population are increasing or decreasing.

When the common ratio is greater than 1, the exponential function is increasing.
When the common ratio is between 0 and 1, the exponential function is decreasing.

$r > 1$, the exponential function is increasing.
$0 < r < 1$, the exponential function is decreasing.