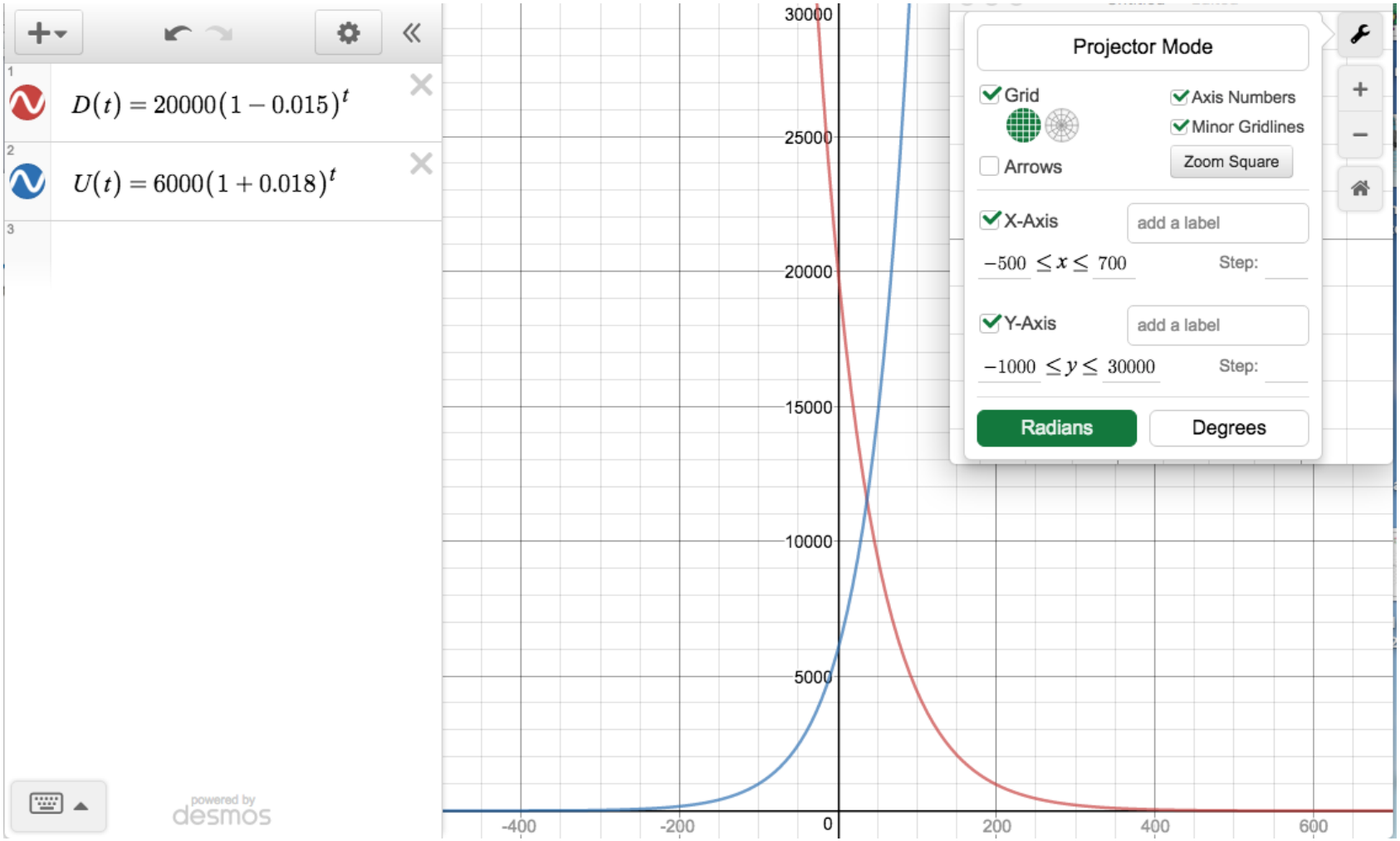
PROBLEM 2 Graphing, Finally!

Let's examine the properties of the graphs of the functions for Downtown and Uptown. Here are the functions again:

Downtown: $D(t) = 20{,}000(1 - 0.015)^t$ Uptown: $U(t) = 6000(1 + 0.018)^t$

Go to Desmos.com.



2. Let's analyze the *y*-intercepts of each function.

a. Identify the *y*-intercepts.

Downtown: y-intercept is 20,000

Uptown: y-intercept is 6,000

b. Interpret the meaning of the *y*-intercept in terms of this problem situation.

Remember, the y-intercept is your starting point. The y-intercept represents the population in each city before any changes occur.

c. Describe how you can determine the *y*-intercept of each function using just the formula for population increase or decrease.

Consider the two functions we just graphed:

$D(t) = 20000(1 - 0.015)^t$

$U(t) = 6000(1 + 0.018)^t$

Exponential functions are written in the standard format:

$f(x) = a \cdot b^x$, where a is the y-intercept and b is the rate of change, also known as the common ratio.

Use Desmos.com

3. Use a graphing calculator to answer each question. Describe your strategy.

a. How long will it take for Downtown's population to be half of what it is now?

Downtown's population = 20,000
20000/2 = 10,000. So, graph the line y = 10,000.
y = 10000 intersects $D(t) = 20000(1 - 0.015)^t$ when $45 < x < 46$.
So, Downtown's population will be half of what it is now in 45 to 46 years.

b. How long will it take for Uptown's population to double from what it is now?

Uptown's population = 6,000
$6000 \times 2 = 12{,}000$. So graph the line y = 12,000.
y = 12000 intersects $U(t) = 6000(1+0.018)^t$ when $38 < x < 39$.
So, Uptown's population will double from what it is now in 38 to 39 years.

c. How many years from now will the populations of Downtown and Uptown be equal? Determine the approximate populations.

Looking at the graph, find the point-of-intersection for the two functions.
The x-values (IQ) = time or the # of years from now
The y-values (DQ) = population
Between 36 and 37 years from now, Downtown and Uptown will have an approximate population of 11,500 each.

Skip to problem 6.

Each population function you graphed has a *horizontal asymptote*. A **horizontal asymptote** is a horizontal line that a function gets closer and closer to, but never intersects.

6. Write the equation for the horizontal asymptote of each population function.

The equation for the horizontal asymptote is y = 0.

7. Does the horizontal asymptote make sense in terms of this problem situation? Explain your reasoning.

Not really. If a population is decreasing, it can eventually be "0". When you are strictly talking about numbers, though, the y-values would get closer, but never reach "0".

8. Identify the domain and range of each function.

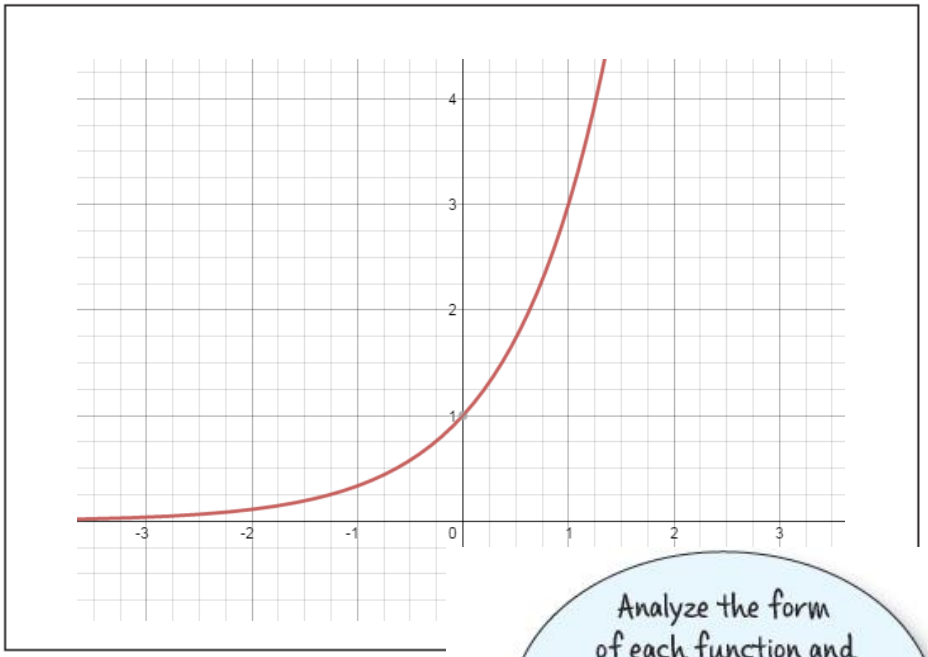
Domain = all real numbers (you can plug in any x-value)
Range = all real numbers greater than zero (y > 0)

PROBLEM 3 The Multiple Representations of Exponentials

1. Complete the table and sketch a graph for each exponential function of the form $f(x) = ab^x$. Then determine the x-intercept(s), y-intercept, asymptote, domain, range, and interval(s) of increase/decrease.

a. $f(x) = 3^x$

x	$f(x)$
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9
3	27



Analyze the form of each function and make a prediction about the shape of the graph before you start. What do the *a* and *b* values tell you?

x-intercept(s): None

y-intercept: (0, 1)

asymptote: $y = 0$

domain: All Real #'s

range: $y > 0$

interval(s) of increase/decrease: Increasing over the entire domain