PROBLEM 2 Horizontal Translations

Consider the three exponential functions shown, where $h(x) = 2^x$ is the basic function.

- $h(x) = 2^x$
- $v(x) = 2^{(x+3)}$
- $w(x) = 2^{(x-3)}$

In Problem 1 *Vertical Translations*, the operations that produced the vertical translations were performed on the function $h(x)$. That is, 3 was added to $h(x)$ and 3 was subtracted from $h(x)$. In this problem, the operations are performed on x, which is the *argument* of the function. The **argument of a function** is the *variable* on which the function operates. So, in this case, 3 is added to x and 3 is subtracted from x.

You can write the given functions $v(x)$ and $w(x)$ in terms of the basic function $h(x)$. To write $v(x)$ in terms of $h(x)$, you just substitute $x + 3$ into the argument for $h(x)$, as shown.

$$h(x) = 2^x$$

$$v(x) = h(x + 3) = 2^{(x+3)}$$

So, $x + 3$ replaces the variable x in the function $h(x) = 2^x$.

1. Write the function $w(x)$ in terms of the basic function $h(x)$.

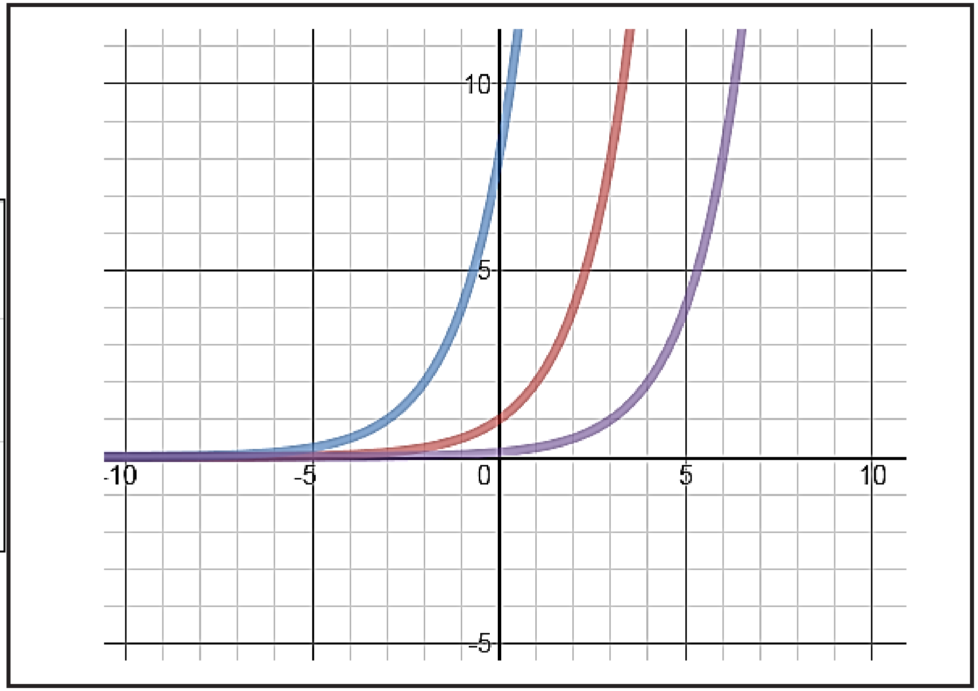
$$w(x) = h(x-3) = 2^{(x-3)}$$

2. Use a graphing calculator to graph each function with the bounds $[-10, 10] \times [-10, 10]$. Then, sketch the graph of each function. Label each graph.

1 $h(x) = 2^x$

2 $v(x) = 2^{(x+3)}$

3 $w(x) = 2^{(x-3)}$



3. Compare the graphs of $v(x)$ and $w(x)$ to the graph of the basic function. What do you notice? This is tricky!!! Look carefully.

The graph of v(x) is shifted to the LEFT 3 units. The graph of w(x) is shifted to the RIGHT 3 units.

4. Write the x-value of each ordered pair for the three given functions. You can use your graphing calculator to determine the x-values.

Use the table feature in Desmos.com to find your x-values.

$h(x) = 2^x$	$v(x) = 2^{(x+3)}$	$w(x) = 2^{(x-3)}$
$(-2, \frac{1}{4})$	$(-5, \frac{1}{4})$	$(1, \frac{1}{4})$
$(-1, \frac{1}{2})$	$(-4, \frac{1}{2})$	$(2, \frac{1}{2})$
$(0, 1)$	$(-3, 1)$	$(3, 1)$
$(1, 2)$	$(-2, 2)$	$(4, 2)$
$(2, 4)$	$(-1, 4)$	$(5, 4)$

5. Use the table to compare the ordered pairs of the graphs of $v(x)$ and $w(x)$ to the ordered pairs of the graph of the basic function $h(x)$. What do you notice?

The y-coordinates stay the same. For v(x), the x-coordinate is 3 more than h(x). For w(x), the x-coordinate is 3 less than h(x).

A horizontal translation of a graph is a shift of the entire graph LEFT or RIGHT. A horizontal translation ***affects the x-coordinate*** of each point on the graph.

Skip #6.

7. Describe each graph in relation to its basic function.

a. Compare $f(x) = b^{x-c}$ to the basic function $h(x) = b^x$ for $c > 0$.

The graph of f(x) is *c* units to the RIGHT of the graph of h(x).

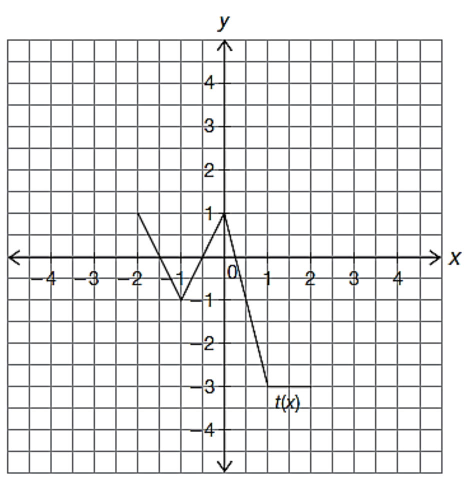
b. Compare $f(x) = b^{x-c}$ to the basic function $h(x) = b^x$ for $c < 0$.

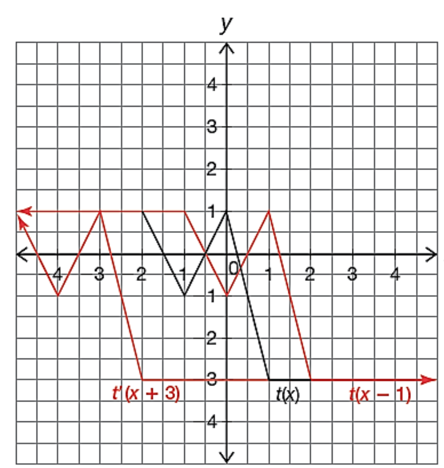
The graph of f(x) is *c* units to the LEFT of the graph of h(x).

8. The graph of a function $t(x)$ is shown. Sketch the graphs of $t'(x)$ and $t''(x)$.

a. $t'(x) = t(x + 3)$

b. $t''(x) = t(x - 1)$





Skip to the table on page 326.

Complete the table by describing the graph of each function as a transformation of its basic function.

Function Form	Equation Information	Description of Transformation of Graph
$f(x) = (x) + b$	$b > 0$	Vertical shift up b units.
	$b < 0$	Vertical shift down b units.
$f(x) = (x - b)$	$b > 0$	Horizontal shift right b units.
	$b < 0$	Horizontal shift left b units.
$f(x) = b^x + k$	$b > 1, k > 0$	Vertical shift up k units.
	$b > 1, k < 0$	Vertical shift down k units.
$f(x) = b^{x-c}$	$b > 1, c > 0$	Horizontal shift right c units.
	$b > 1, c < 0$	Horizontal shift left c units.