5.3

Let the Transformations Begin!

Translations of Linear and Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Translate linear and exponential functions vertically.
- Translate linear and exponential functions horizontally.

KEY TERMS

- basic function
- transformation
- vertical translation
- coordinate notation
- argument of a function
- horizontal translation

PROBLEM 1 Vertical Translations

Consider the three linear functions shown.

- $g(x) = x$
- $c(x) = (x) + 3$
- $d(x) = (x) - 3$

The first function is the *basic function*. A **basic function** is the simplest function of its type. In this case, $g(x) = x$ is the simplest linear function. It is in the form $f(x) = ax + b$, where $a = 1$ and $b = 0$.

You can write the given functions $c(x)$ and $d(x)$ in terms of the basic function $g(x)$. For example, because $g(x) = x$, you can substitute $g(x)$ for x in the equation for $c(x)$, as shown.

$$c(x) = (x) + 3$$
$$\downarrow$$
$$c(x) = g(x) + 3$$

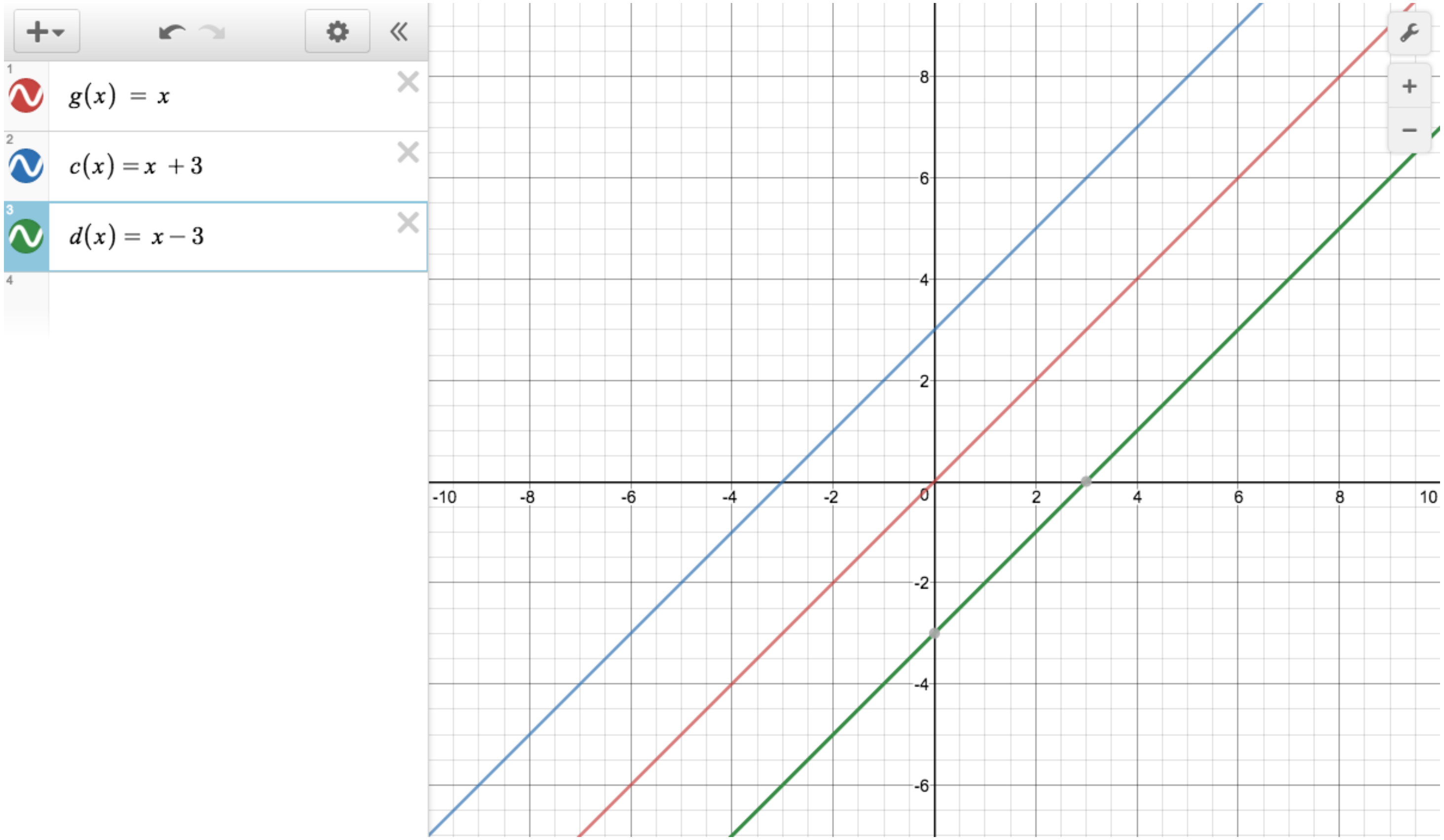
1. Write the function $d(x)$ in terms of the basic function $g(x)$.

   $d(x) =$ $g(x) - 3$

2. Describe the operation performed on the basic function $g(x)$ to result in each of the equations for $c(x)$ and $d(x)$.

   For $c(x)$, the constant, 3, is added to $g(x)$. For $d(x)$, 3 is subtracted from $g(x)$.

3. Use Desmos.com to graph each function: $g(x)$, $c(x)$, and $d(x)$.



4. Compare the y-intercepts of the graphs of $c(x)$ and $d(x)$ to the y-intercept of the basic function $g(x)$. What do you notice?

For $c(x)$, move the y-intercept of $g(x)$ UP 3 units. For $d(x)$, move the y-intercept of $g(x)$ DOWN 3 units.

5. Write the y-value of each ordered pair for the three given functions.

$g(x) = x$	$c(x) = (x) + 3$	$d(x) = (x) - 3$
$(-2, \underline{-2})$	$(-2, \underline{1})$	$(-2, \underline{-5})$
$(-1, \underline{-1})$	$(-1, \underline{2})$	$(-1, \underline{-4})$
$(0, \underline{0})$	$(0, \underline{3})$	$(0, \underline{-3})$
$(1, \underline{1})$	$(1, \underline{4})$	$(1, \underline{-2})$
$(2, \underline{2})$	$(2, \underline{5})$	$(2, \underline{-1})$

6. Use the table to compare the ordered pairs of the graphs of $c(x)$ and $d(x)$ to the ordered pairs of the graph of the basic function $g(x)$. What do you notice?

The x-coordinates never change. For $c(x)$, each y-coordinate is 3 more than the y-coordinate of $g(x)$. For $d(x)$, each y-coordinate is 3 less than the y-coordinate of $g(x)$.

A vertical translation is a type of transformation that shifts the entire graph **UP** or **DOWN.** A vertical translation ***affects the y-coordinate*** of each point on the graph.

A vertical shift occurs when a number is added to or subtracted from the whole basic function!

Now, let's consider the three exponential functions shown.

- $h(x) = 2^x$
- $s(x) = (2^x) + 3$
- $t(x) = (2^x) - 3$

In this case, $h(x) = 2^x$ is the basic function because it is the simplest exponential function with a base of 2. It is in the form $f(x) = a \cdot b^x$, where $a = 1$ and $b = 2$.

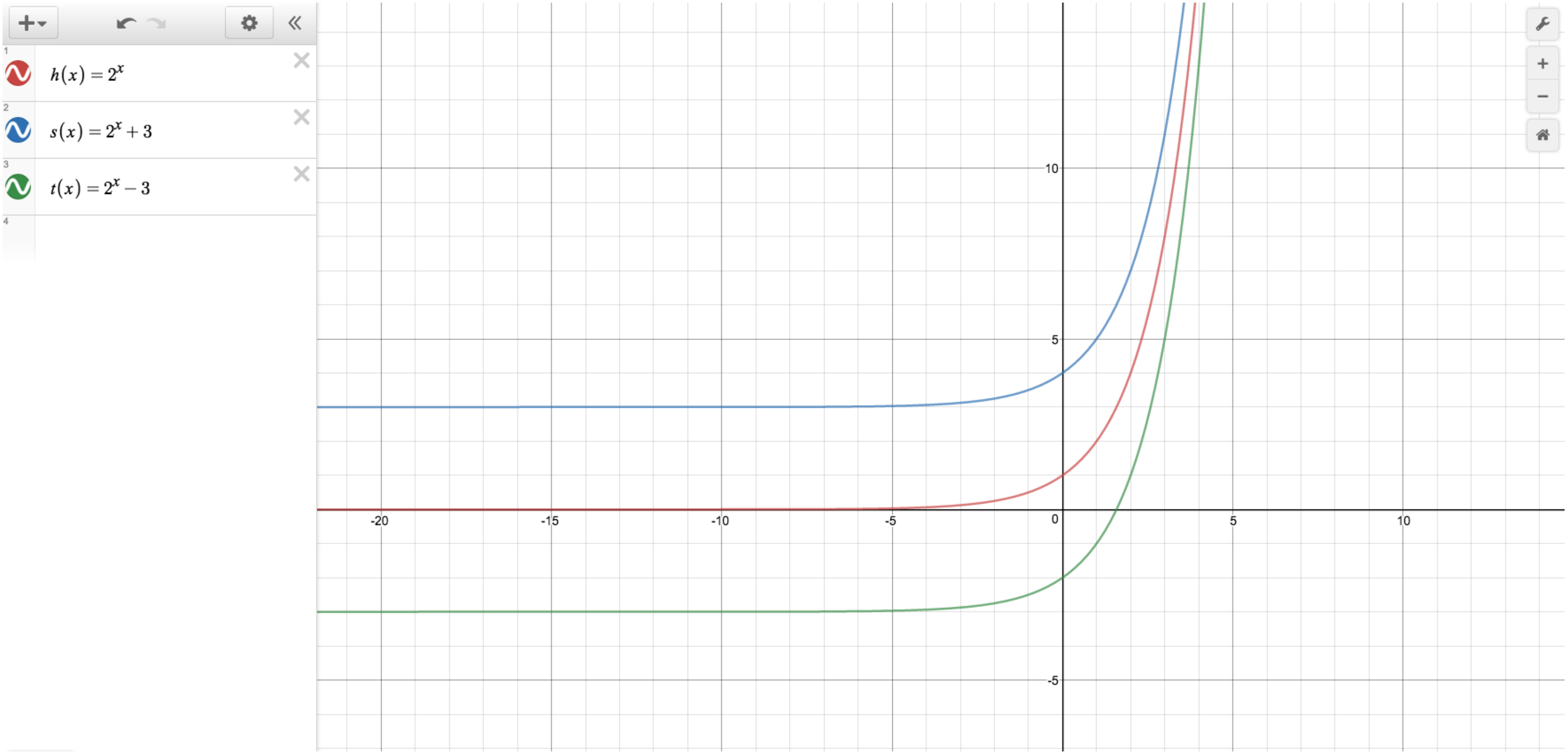
8. Write the functions $s(x)$ and $t(x)$ in terms of the basic function $h(x)$. Then, describe the operation performed on the basic function $h(x)$ to result in each of the equations for $s(x)$ and $t(x)$.

   $s(x) =$ $h(x) + 3$

   $t(x) =$ $h(x) - 3$

   For $s(x)$, the constant, 3, is added to $h(x)$. For $t(x)$, 3 is subtracted from $h(x)$.

9. Use Desmos.com to graph each function: *h*(x), s(*x*), and *t*(*x*).



10. Compare the y-intercepts of the graphs of *s*(*x*) and *t*(*x*) to the y-intercept of the basic function *h*(*x*). What do you notice? Are the results the same as when you compared the graphs of the linear functions in Question 4?

For *s*(*x*), move the graph of *h*(*x*) UP 3 units. For *t*(*x*), move the graph of *h*(*x*) DOWN 3 units. Yes, the results are the same as the linear function graphs.

11. Write the y-value of each ordered pair for the three given functions.

$h(x) = 2^x$	$s(x) = (2^x) + 3$	$t(x) = (2^x) - 3$
$(-2, \frac{1}{4})$	$(-2, \frac{13}{4})$	$(-2, -\frac{11}{4})$
$(-1, \frac{1}{2})$	$(-1, \frac{7}{2})$	$(-1, -\frac{5}{2})$
(0, 1)	(0, 4)	(0, −2)
(1, 2)	(1, 5)	(1, −1)
(2, 4)	(2, 7)	(2, 1)

12. Use the table to compare the ordered pairs of the graphs of $s(x)$ and $t(x)$ to the ordered pairs of the graph of the basic function $h(x)$. What do you notice? Are the results the same as when you compared the y-values for the linear functions in Question 6?

The x-coordinates never change. For $s(x)$, each y-coordinate is 3 more than the y-coordinate of $h(x)$. For $t(x)$, each y-coordinate is 3 less than the y-coordinate of $h(x)$.

Yes, the results are the same as the y-values for the linear functions.

13. Explain how you know that the graphs of $s(x)$ and $t(x)$ are vertical translations of the graph of $h(x)$.

Every point on the graph of $s(x)$ is 3 units UP from the graph of $h(x)$. Every point on the graph of $t(x)$ is 3 units DOWN from the graph of $h(x)$.