5.4

Take Some Time to Reflect

Reflections of Linear and Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Reflect linear and exponential functions vertically.
- Reflect linear and exponential functions horizontally.
- Determine characteristics of graphs after transformations.

KEY TERMS

- reflection
- line of reflection

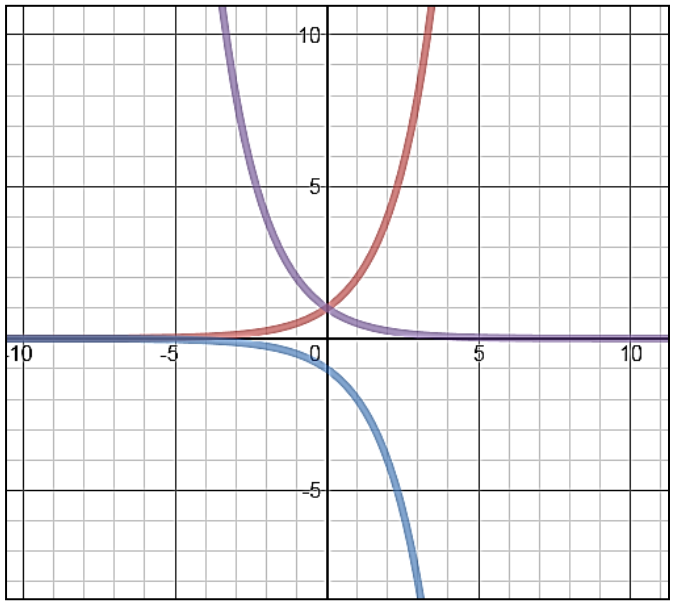
PROBLEM 1 Reflections

Consider the three exponential functions shown, where $h(x) = 2^x$ is the basic function.

- $h(x) = 2^x$
- $m(x) = -(2^x)$
- $n(x) = 2^{(-x)}$

Skip to #4. Using Desmos.com to graph the three exponential functions, sketch the graph of each function. Label each graph.

1. $h(x) = 2^x$
2. $m(x) = -\left(2^x\right)$
3. $n(x) = 2^{(-x)}$



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5. Compare the graphs of $m(x)$ and $n(x)$ to the graph of the basic function $h(x)$. What do you notice?

$m(x) = -(2^x)$ is reflected over the x-axis (or the line y = 0).
$n(x) = 2^{(-x)}$ is reflected over the y-axis (or the line x = 0).

6. Complete the table of ordered pairs for the three given functions.

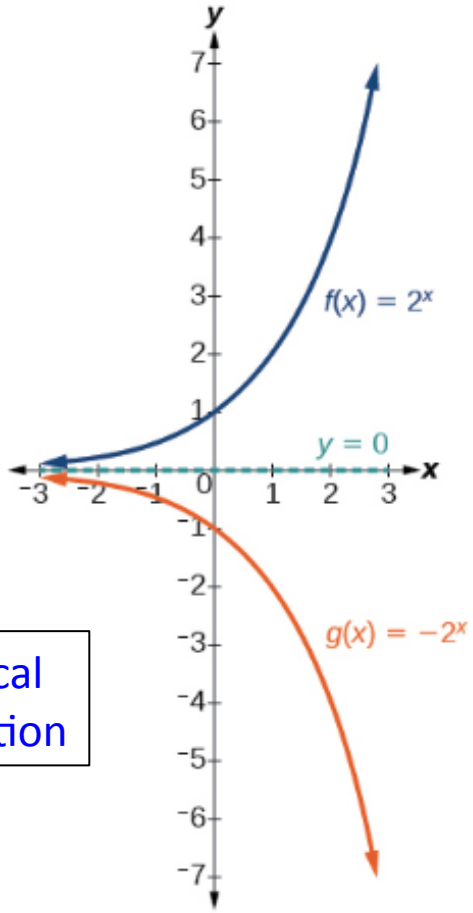
$h(x) = 2^x$	$m(x) = -(2^x)$	$n(x) = 2^{(-x)}$
$(-2, \frac{1}{4})$	$(-2, \underline{-\frac{1}{4}})$	$(\underline{2}, \frac{1}{4})$
$(-1, \frac{1}{2})$	$(-1, \underline{-\frac{1}{2}})$	$(\underline{1}, \frac{1}{2})$
$(0, 1)$	$(0, \underline{-1})$	$(\underline{0}, 1)$
$(1, 2)$	$(1, \underline{-2})$	$(\underline{-1}, 2)$
$(2, 4)$	$(2, \underline{-4})$	$(\underline{-2}, 4)$

7. Use the table to compare the ordered pairs of the graphs of $m(x)$ and $n(x)$ to the ordered pairs of the graph of the basic function $h(x)$. What do you notice?

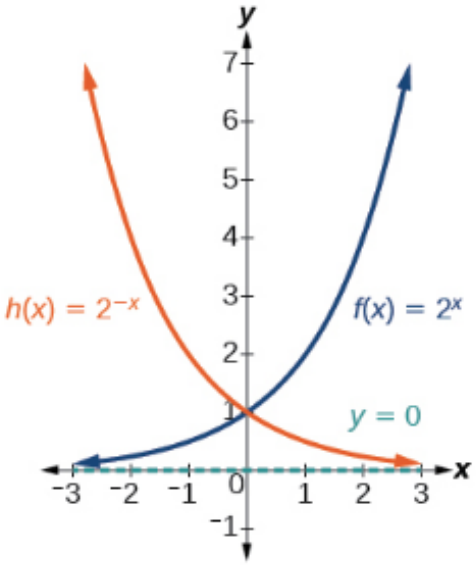
Comparing $h(x)$ and $m(x)$, **the y-coordinates have opposite signs**. Comparing $h(x)$ and $n(x)$, **the x-coordinates have opposite signs**.

A **reflection** of a graph is a mirror image of the graph about a *line of reflection*. A **line of reflection** is the line that the graph is reflected about. A horizontal line of reflection affects the *y*-coordinates, and a vertical line of reflection affects the *x*-coordinates.

Reflection about the *x*-axis

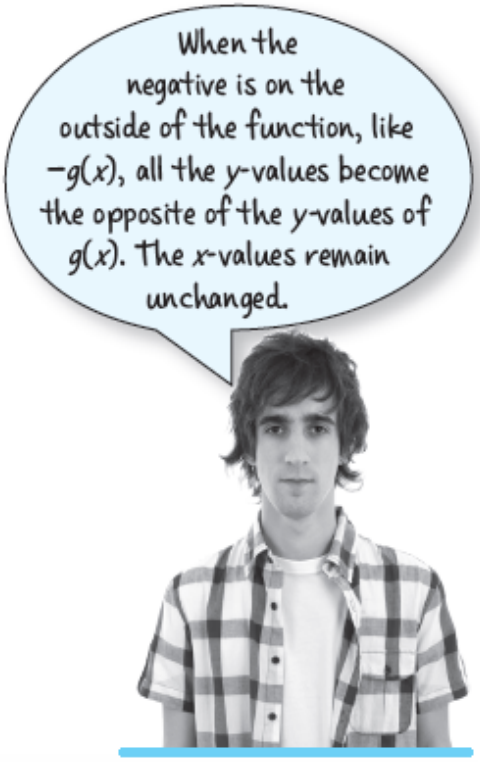


Reflection about the *y*-axis



Vertical Reflection

Horizontal Reflection



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9. Describe each graph in relation to its basic function.

a. Compare $f(x) = -(b^x)$ to the basic function $h(x) = b^x$.

The graph is reflected over the x-axis or the line y = 0.
It is a vertical reflection because the graph flips up or down so the y-coordinates change signs (+/-).

b. Compare $f(x) = b^{(-x)}$ to the basic function $h(x) = b^x$.

The graph is reflected over the y-axis or the line x = 0.
It is a horizontal reflection because the graph flips left or right so the x-coordinates change signs (+/-).

Skip to #11.

11. Write the equation of each function after a reflection about the horizontal line $y = 0$. Then, write the equation after a reflection about the vertical line $x = 0$.

a. $a(x) = 5^x$

Reflection about $y = 0$: $a'(x) =$ -5^x

Reflection about $x = 0$: $a''(x) =$ 5^{-x}

b. $b(x) = -2x^2$

Reflection about $y = 0$: $b'(x) =$ $-(-2)x^2 = 2x^2$

Reflection about $x = 0$: $b''(x) =$ $-2(-x)^2 = -2x^2$

c. $c(x) = \frac{5}{4}x^3$

Reflection about $y = 0$: $c'(x) =$ $-\frac{5}{4}x^3$

Reflection about $x = 0$: $c''(x) =$ $\frac{5}{4}(-x)^3$