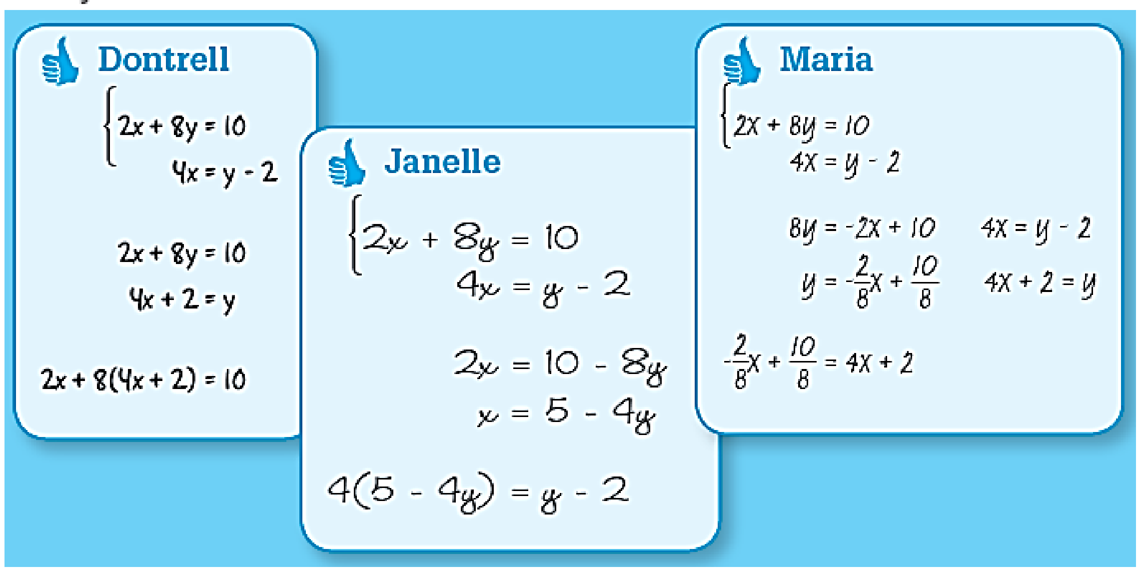
PROBLEM 3 Transforming Equations: More Than Meets the Eye

Not all systems will be written in slope-intercept form or function notation. Systems can also be written in standard form. Let's explore a system in standard form.

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

Do you think there is more than one way to transform one of the equations in the system to create a new equation with only one unknown?

1. Analyze each student's work.



a. Describe the method Dontrell used to solve this system of equations and explain why he is correct.

He solved for *y* in the 2nd equation. Then, substituted the *y*-value for *y* in the 1st equation.

b. Describe the method Janelle used to solve this system of equations and explain why her reasoning is correct.

She solved for *x* in the 1st equation. Then, substituted the *x*-value for *x* in the 2nd equation.

c. Describe the method Maria used to solve this system of equations and explain why her reasoning is correct.

She solved for *y* in both equations. Then, set the 2 equations equal to each other.

2. Which method do you prefer for solving this system of equations?

I prefer Dontrell's method because it was easy to solve for *y* in the 2nd equation.

I prefer Janelle's method because it is easily repeatable. Solve for *x* in the 1st equation, then plug the *x*-value into the 2nd equation.

I prefer Maria's method because it easy to keep track of what I am doing. Solve for *y* each time, then set the *y*-values equal to each other.

3. Use one of the methods shown or use your own method to determine the solution to this system of equations.

Methods will vary.

$$2x + 8y = 10$$
$$4x + 2 = y$$

$$2x + 8(4x + 2) = 10$$
$$2x + 32x + 16 = 10$$

$$\frac{34x}{34} = \frac{-6}{34}$$
$$x = \frac{-6}{34} = \frac{-3}{17}$$

$$4\left(-\frac{3}{17}\right) + 2 = y$$
$$-\frac{12}{17} + 2 = y$$
$$-\frac{12}{17} + \frac{34}{17} = y$$
$$\frac{22}{17} = y$$

The solution is $\left(-\frac{3}{17}, \frac{22}{17}\right)$.

4. Soo Jin encountered this system of linear equations.

$$\begin{cases} 3.5x + 1.2y = 8 \\ 4.7x + 0.3y = 10.3 \end{cases}$$

However, Soo Jin has decimaphobia—a fear of decimals! Sammy tells her she has nothing to fear. He says, "All you need to do is multiply each equation by 10 to transform the system into whole numbers."

a. Is Sammy correct? Explain why or why not.

Sammy is correct. Multiplying every term in both equations by 10 changes the decimals to whole numbers. But, the solution stays the same.

b. Soo Jin attempts Sammy's method. Her work is shown.

Soo Jin

$$\begin{cases} 35x + 12y = 8 \\ 47x + 3y = 103 \end{cases}$$

Explain the mistake(s) Soo Jin made and then determine the correct way to rewrite this system.

Soo Jin forgot to multiply the "8" by 10.
The 1st equation should be $35x + 12y = \mathbf{80}$.
The 2nd equation is correct.
The correct way to write this system is:

$$\begin{cases} 35x + 12y = 80 \\ 47x + 3y = 103 \end{cases}$$

Talk the Talk

1. Use any method of substitution to determine the solutions for each of the systems of linear equations.

a. $\begin{cases} 8x - 2y = 7 \\ 2x + y = 4 \end{cases}$

b. $\begin{cases} 0.4x + 0.3y = 1 \\ 0.1y = 0.2x \end{cases}$

Get *y* by itself.

a.

$2x + y = 4$

$y = 4 - 2x$

$8x - 2(4 - 2x) = 7$

$8x - 8 + 4x = 7$

$12x - 8 = 7$

$12x = 15$

$x = 1.25$

b.

$4x + 3y = 10$

$y = 2x$

$4x + 3(2x) = 10$

$4x + 6x = 10$

$10x = 10$

$x = 1$

Plug in the *x*-value & solve for *y*.

a.

$2(1.25) + y = 4$

$2.5 + y = 4$

$y = 1.5$

The solution is (1.25, 1.5).

b.

$y = 2(1)$

$y = 2$

The solution is (1, 2).

c. $\begin{cases} \frac{1}{2}x + \frac{1}{4}y = 6 \\ y = 4 \end{cases}$

$$4\left(\frac{1}{2}x + \frac{1}{4}y\right) = 6(4)$$

$$2x + y = 24$$

$$2x + (4) = 24$$

$$2x = 20$$

$$x = 10$$

The solution is (10, 4).

d. $\begin{cases} 6x + 3y = 5 \\ y = -2x + 1 \end{cases}$

$$6x + 3(-2x + 1) = 5$$

$$6x - 6x + 3 = 5$$

$$3 \neq 5$$

There is no solution.

A system of equations may have one unique solution, infinitely many solutions, or no solution. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.

2. Complete the table.

	Consistent Systems		**Inconsistent Systems**
Number of Solutions	One Unique Solution	Infinitely Many Solutions	No Solution
Description of *y*-intercepts	*y*-intercepts can be the same or different	*y*-intercepts are the same	*y*-intercepts are different
Description of Graph	Lines intersect	Lines are the same	Lines are parallel