

6.2

There's Another Way?

Using Linear Combinations to Solve a Linear System

LEARNING GOALS

In this lesson, you will:

- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using linear combinations (elimination).

KEY TERM

- linear combinations method

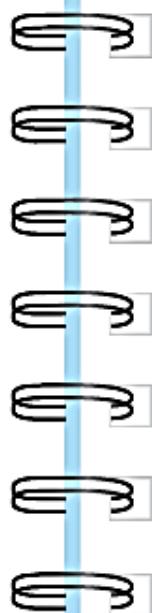
PROBLEM 3 Linear Combinations

Skip to Page 388



The algebraic method you used to solve the linear systems in Problems 1 and 2 is called the *linear combinations method*. The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

In many cases, one or both of the equations in the system must be multiplied by a constant so that when the equations are added together, the result is an equation in one variable. This means that the coefficients of either the term containing x or y must be opposites.



Let's consider a system where neither of the x - or y -terms are opposites.

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

The equations are in standard form.

Multiply each equation by a constant that results in opposite coefficients for one of the variables.

$$\rightarrow 3(4x + 2y) = 3(3)$$

$$-2(5x + 3y) = -2(4)$$

$$12x + 6y = 9$$

$$-10x - 6y = -8$$

Now that the y -values are opposites, you can solve this linear system.



1. Solve the new linear system shown in the worked example.

$$\begin{array}{r} 12x + 6y = 9 \\ -10x - 6y = -8 \\ \hline 2x = 1 \\ x = \frac{1}{2} \end{array}$$

$$\begin{array}{r} 4\left(\frac{1}{2}\right) + 2y = 3 \\ 2 + 2y = 3 \\ 2y = 1 \\ y = \frac{1}{2} \end{array}$$

You can check your solution by substituting the ordered pair back into the original equations.



2. Describe the first step needed to solve each system using the linear combination method. Identify the variable that will be solved when you add the equations.

a. $5x + 2y = 10$ and $3x + 2y = 6$

Look for variables with the same coefficients.

Multiply the second equation by -1 to make the y 's cancel.

b. $x + 3y = 15$ and $5x + 2y = 7$

Try to multiply only one equation by a constant.

Multiply the first equation by -5 to make the x 's cancel.

c. $4x + 3y = 12$ and $3x + 2y = 4$

Multiply the first equation by 3 and the second equation by -4 to make the x 's cancel.

Are there other ways to create opposite coefficients for either variable?





3. Solve each system using linear combinations.

a.
$$\begin{cases} 2x + y = 8 \\ 3x - y = 7 \end{cases}$$

$$\begin{array}{r} 2x + \cancel{y} = 8 \\ 3x - \cancel{y} = 7 \\ \hline 5x = 15 \\ x = 3 \end{array} \qquad \begin{array}{l} 2(3) + y = 8 \\ 6 + y = 8 \\ y = 2 \end{array}$$

The solution is (3, 2).

b.
$$\begin{cases} 4x + 3y = 24 \\ 3x + y = -2 \end{cases}$$

$$\begin{array}{r} 4x + 3y = 24 \\ -3(3x + y = -2) \\ \hline 4x + 3y = 24 \\ -9x - 3y = 6 \\ \hline -5x = 30 \\ x = -6 \end{array} \qquad \begin{array}{l} 4(-6) + 3y = 24 \\ -24 + 3y = 24 \\ 3y = 48 \\ y = 16 \end{array}$$

The solution is (-6, 16).

Times 2 \rightarrow
$$\begin{cases} 3x + 5y = 17 \\ 2x + 3y = 11 \end{cases}$$

 Times -3 \rightarrow

$$\begin{array}{r} 6x + 10y = 34 \\ -6x - 9y = -33 \\ \hline y = 1 \end{array} \qquad \begin{array}{l} 3x + 5(1) = 17 \\ 3x + 5 = 17 \\ 3x = 12 \\ x = 4 \end{array}$$

The solution is (4, 1).

Add this problem to Page 389.

Solve this system using Linear Combinations/Elimination method.

$$-3x - 2y = 1$$

$$12x + 8y = 12$$

Multiply the first equation by 4

$$4(-3x - 2y = 1)$$

$$12x + 8y = 12$$

$$~~-12x - 8y = 4~~$$

$$12x + 8y = 12$$

$$0 = 16$$

There is no solution.
This system is inconsistent.

Go to Page 384

PROBLEM 1 People Love Their Comics—Even On-Line!



There are a total of 324 people who joined the Comic Gurus group on a social media site. Female group members outnumber males by 34. Determine how many males and females joined the Comic Gurus group.

1. Write an equation in standard form that represents the total number of people who joined the Comic Gurus group. Use x to represent the female members, and use y to represent the male members.

$$x + y = 324$$

2. Write an equation in standard form to represent the number of female members in relationship to the number of male members.

$$x = y + 34 \quad \text{or} \quad x - y = 34$$

$$\text{System: } x + y = 324$$

$$x - y = 34$$

Go to Page 386

PROBLEM 2 Let It Snow . . . For Winter Get-Aways



Let It Snow Resort offers two winter specials: the Get-Away Special and the Extended Stay Special. Let It Snow claims that the Extended Stay Special is the better deal. The Get-Away Special offers two nights of lodging and four meals for \$270. The Extended Stay Special offers three nights of lodging and eight meals for \$435. Determine if the Extended Stay Special is the better deal.

1. Write an equation in standard form that represents the Get-Away Special. Let n represent the cost for one night of lodging at the resort, and let m represent the cost for each meal.

$$2n + 4m = 270$$

2. Write an equation in standard form that represents the Extended Stay Special. Use the same variables you used in Question 1.

$$3n + 8m = 435$$

$$\text{System: } 2n + 4m = 270$$

$$3n + 8m = 435$$