Solve Using the Elimination Method

1. $\begin{cases} 5x+3y=16 \\ -10x-6y=-22 \end{cases}$

$$\begin{array}{r} 10x+6y=32 \\ -10x-6y=-22 \\ \hline 0\neq 10 \end{array}$$

No Solution

2. $\begin{cases} 7x-7y=14 \\ 14x-14y=28 \end{cases}$

$$\begin{array}{r} -14x+14y=-28 \\ 14x-14y=28 \\ \hline 0=0 \end{array}$$

Infinite Solutions

6.3

What's for Lunch?

Solving More Systems

LEARNING GOALS

In this lesson, you will:

- Write a linear system of equations to represent a problem context.
- Solve a linear system of equations using the linear combinations method.

PROBLEM 1 What's On the Menu Today?

Constance owns a small lunch cart. She changes her menu daily. Yesterday, she offered a chef salad for $5.75 or a hoagie for $5.00. She sold 85 lunches for a total of $464. Determine how many chef salads and hoagies she sold.

1. Write an equation in standard form that represents the total number of lunches in terms of the number of chef salads sold and the number of hoagies sold. Let *x* represent the number of chef salads sold, and let *y* represent the number of hoagies sold.

   $x + y = 85$

2. Write an equation in standard form that represents the amount of money collected. Use the same variables as those used in Question 1.

   $5.75x + 5y = 464$

3. Write a system of linear equations to represent this problem situation.

   $$\begin{cases} x + y = 85 \\ 5.75x + 5y = 464 \end{cases}$$

4. What methods can you use to solve this system of linear equations?

   Graphing, substitution, or linear combinations

5. Determine the solution of this linear system of equations by using linear combinations. Then, check your answer.

$$-5(x + y = 85)$$
$$-5x - 5y = -425$$
$$5.75x + 5y = 464$$
$$0.75x = 39$$
$$x = 52$$

$$52 + y = 85$$
$$y = 33$$

The solution is (52, 33).

Check: $52 + 33 = 85$

$85 = 85$ ✓

$5.75(52) + 5(33) = 464$

$299 + 165 = 464$

$464 = 464$ ✓

6. Interpret your solution to the linear system in terms of this problem situation.

Constance sold 52 chef salads and 33 hoagies.

PROBLEM 2 You're My Best Buddy!

The School Spirit Club is making beaded friendship bracelets with the school colors to sell in the school store. The bracelets are black and orange and come in two lengths: 5 inches and 7 inches. The club has enough beads to make a total of 84 bracelets. So far, they have made 49 bracelets, which represents $\frac{1}{2}$ the number of 5-inch bracelets plus $\frac{3}{4}$ the number of 7-inch bracelets they plan to make and sell. Determine how many 5-inch and 7-inch bracelets the club plans to make.

1. Write an equation in standard form that represents the total number of bracelets the School Spirit Club can make out of the beads that they have. Let *x* represent the number of 5-inch bracelets, and let *y* represent the number of 7-inch bracelets.

$$x + y = 84$$

2. Write an equation in standard form that represents the number of bracelets the School Spirit Club has made so far. Use the same variables as those used in Question 1.

$$\frac{1}{2}x + \frac{3}{4}y = 49$$

3. Write a system of linear equations that represents this problem situation.

$$\begin{cases} x + y = 84 \\ \frac{1}{2}x + \frac{3}{4}y = 49 \end{cases}$$

4. Karyn says that the first step she would take to solve this system would be to first multiply the second equation by the least common denominator (LCD) of the fractions. Is she correct? Explain your reasoning.
   Yes. Clear the fractions! The LCD of 2 and 4 is **4**.

5. Rewrite the equation containing fractions as an equivalent equation without fractions.

$$4\left(\frac{1}{2}x+\frac{3}{4}y=49\right) \longrightarrow 2x+3y=196$$

6. Determine the solution to the system of equations by using linear combinations and check your answer.

$$\begin{cases} x+y=84 \\ 2x+3y=196 \end{cases}$$

$$-2(x+y=84)$$
$$-2x-2y=-168$$
$$2x+3y=196$$
$$y=28$$

$$x+28=84$$
$$x=56$$

Check: $56+28=84$
$84=84$ ✓

$$\frac{1}{2}(56)+\frac{3}{4}(28)=49$$
$$28+21=49$$
$$49=49$$ ✓

7. Interpret the solution of the linear system in terms of this problem situation.
   The School Spirit Club plans to make a total of 56 5-inch and 28 7-inch bracelets.

Talk the Talk

1. Solve each linear system using linear combinations. Check all solutions.

Which variable has a coefficient of 1?

a. $\begin{cases} x + 2y = 2 \\ 5x - 3y = -29 \end{cases}$

$$\begin{aligned} -5(x + 2y &= 2) \\ -5x - 10y &= -10 \\ 5x - 3y &= -29 \\ \hline -13y &= -39 \\ y &= 3 \end{aligned}$$

$$\begin{aligned} x + 2(3) &= 2 \\ x + 6 &= 2 \\ x &= -4 \end{aligned}$$

The solution is (−4, 3).

Check: $-4 + 2(3) = 2$
$-4 + 6 = 2$
$2 = 2$ ✓

$5(-4) - 3(3) = -29$
$-20 - 9 = -29$
$-29 = -29$ ✓

Clear the fractions!

b. $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 3 \\ 3x + 5y = 36 \end{cases}$

$$\begin{aligned} -6\left(\tfrac{1}{2}x + \tfrac{1}{3}y = 3\right) \\ -3x - 2y &= -18 \\ 3x + 5y &= 36 \\ \hline 3y &= 18 \\ y &= 6 \end{aligned}$$

$$\begin{aligned} 3x + 5(6) &= 36 \\ 3x + 30 &= 36 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

The solution is (2, 6).

Check: $\frac{1}{2}(2) + \frac{1}{3}(6) = 3$
$1 + 2 = 3$
$3 = 3$ ✓

$3(2) + 5(6) = 36$
$6 + 30 = 36$
$36 = 36$ ✓

c. $\begin{cases} 0.6x + 0.2y = 2.2 \\ 0.5x - 0.2y = 1.1 \end{cases}$

$$\begin{aligned} 0.6x + 0.2y &= 2.2 \\ 0.5x - 0.2y &= 1.1 \\ \hline 1.1x &= 3.3 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 0.6(3) + 0.2y &= 2.2 \\ 1.8 + 0.2y &= 2.2 \\ 0.2y &= 0.4 \\ y &= 2 \end{aligned}$$

The solution is (3, 2).

Clear the decimals!

Check: $0.6(3) + 0.2(2) = 2.2$
$1.8 + 0.4 = 2.2$
$2.2 = 2.2$ ✓

$0.5(3) - 0.2(2) = 1.1$
$1.5 - 0.4 = 1.1$
$1.1 = 1.1$ ✓

d. $\begin{cases} \frac{1}{2}x + \frac{3}{5}y = 17 \\ \frac{1}{5}x + \frac{3}{4}y = 17 \end{cases}$

$10\left(\frac{1}{2}x + \frac{3}{5}y = 17\right)$

$20\left(\frac{1}{5}x + \frac{3}{4}y = 17\right)$

$$\begin{aligned} 5x + 6y &= 170 \\ 4x + 15y &= 340 \end{aligned}$$

$-4(5x + 6y = 170)$

$5(4x + 15y = 340)$

$$\begin{aligned} -20x - 24y &= -680 \\ 20x + 75y &= 1700 \\ \hline 51y &= 1020 \\ y &= 20 \end{aligned}$$

$$\begin{aligned} 5x + 6(20) &= 170 \\ 5x + 120 &= 170 \\ 5x &= 50 \\ x &= 10 \end{aligned}$$

The solution is (10, 20).

Clear the fractions!

Check: $\frac{1}{2}(10) + \frac{3}{5}(20) = 17$
$5 + 12 = 17$
$17 = 17$ ✓

$\frac{1}{5}(10) + \frac{3}{4}(20) = 17$
$2 + 15 = 17$
$17 = 17$ ✓

2. You have used three different methods for solving systems of equations: graphing, substitution, and linear combinations. Describe how to use each method and the characteristics of the system that makes this method most appropriate.

- Graphing Method:

Graph each equation. The solution is the point-of-intersection.
This method is best when the numbers are easy to graph, or you need an estimate. Graphing is also used to predict future results.

- Substitution Method:

Isolate the variable in one equation and substitute its value or expression into the other equation. Then, solve for each variable.
This method is best when it's easy to isolate one of the variables, i.e. the equations are written in slope-intercept form.

- Linear Combinations Method:

Combine the equations $(+, -, \times)$ to eliminate one of the variables. Then, solve for each variable.
This method is best when the coefficients of a variable are the same or opposites, or the equations are written in standard form.